The Hall effect in RAMSES for star formation

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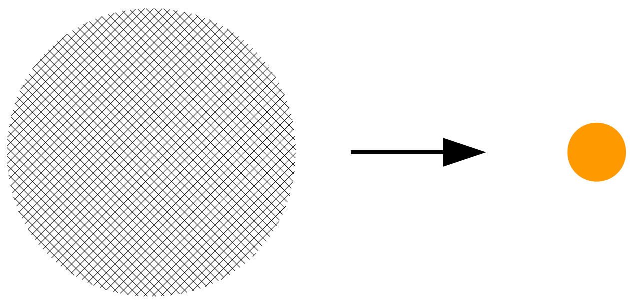
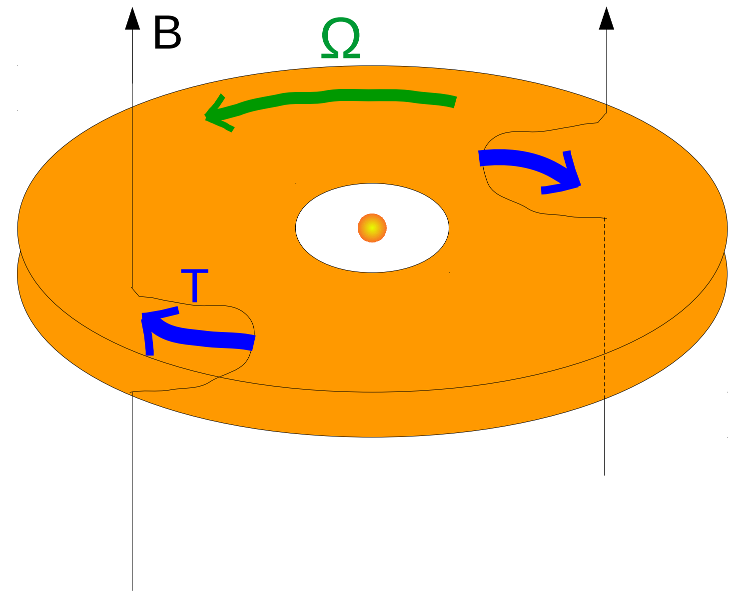
DENSE CORE COLLAPSE



$l_c \approx 10^{21} cm^2 s^{-1}$ $\qquad$ $l_c \approx 10^{18} cm^2 s^{-1}$

Loss of angular momentum

MAGNETIC BRAKING



NON IDEAL MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot [\rho \mathbf{u}] = 0,$$
$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \left[\rho \mathbf{u}\mathbf{u} + \left(P + \frac{B^2}{2}\right)\mathbb{I} - \mathbf{B}\mathbf{B}\right] = -\rho \nabla \Phi,$$
$$\frac{\partial E_{\rm tot}}{\partial t} + \nabla \cdot [(E_{\rm tot} + P_{\rm tot})\mathbf{u} - (\mathbf{u}\cdot\mathbf{B})\mathbf{B} - \mathbf{E}_{\rm NIMHD} \times \mathbf{B}] = -\rho \mathbf{u} \cdot \nabla \Phi,$$
$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u} \times \mathbf{B} + \mathbf{E}_{\rm NIMHD}) = 0,$$
$$\nabla \cdot \mathbf{B} = 0.$$

Ambipolar and Ohmic diffusions regulate the magnetic braking and the accumulation of magnetic flux

e.g. *Mellon & Li 09, Duffin & Pudritz 09, Kunz & Mouschovias 10, Machida+11, Tomida+15, Masson+16*

$$\mathbf{E}_{\rm NIMHD} = \underbrace{-\eta_\Omega \nabla \times \mathbf{J}}_{\text{Ohmic diffusion}} - \underbrace{\eta_{\rm H}(\mathbf{J}) \times \mathbf{B}}_{\text{Hall effect}} + \underbrace{\eta_{\rm AD}(\mathbf{J} \times \mathbf{B}) \times \mathbf{B}}_{\text{Ambipolar diffusion}}$$

Ohmic diffusion $\quad$ Hall effect $\quad$ Ambipolar diffusion

See *Masson+12* for the implementation of OD and AD in RAMSES

IN STAR FORMATION

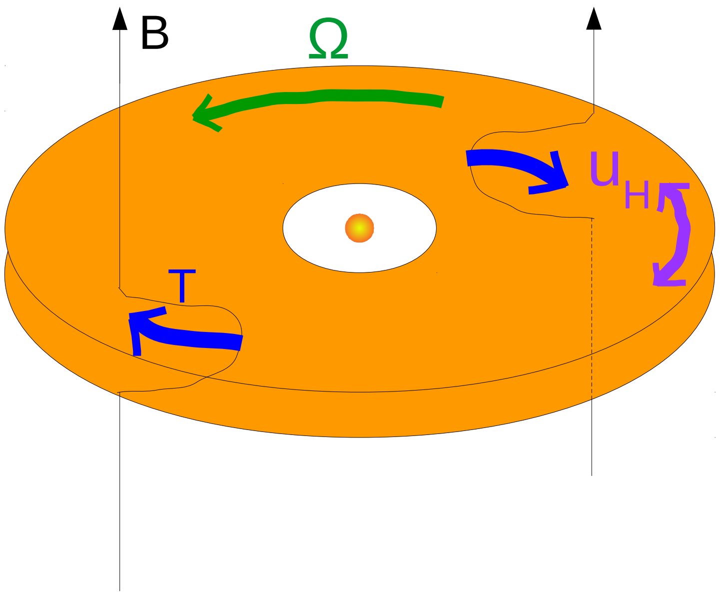
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\mathbf{u} \times \mathbf{B} - \frac{\eta_\mathrm{H}}{||\mathbf{B}||}\mathbf{J} \times \mathbf{B}\right]$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[(\mathbf{u} + \mathbf{u}_\mathrm{H}) \times \mathbf{B}\right]$$

$$\mathbf{u}_\mathrm{H} = -\eta_\mathrm{H}\frac{\mathbf{J}}{||\mathbf{B}||}$$

Hall speed : motion of the field lines

The Hall effect generates rotation in either direction



PERTURBATIVE ANALYSIS

Dispersion relation

$$\omega = \pm\frac{\eta_\mathrm{H} k^2}{2} + \sqrt{\left(\frac{\eta_\mathrm{H} k^2}{2}\right)^2 + k^2 c_\mathrm{A}^2}$$

Wave speed

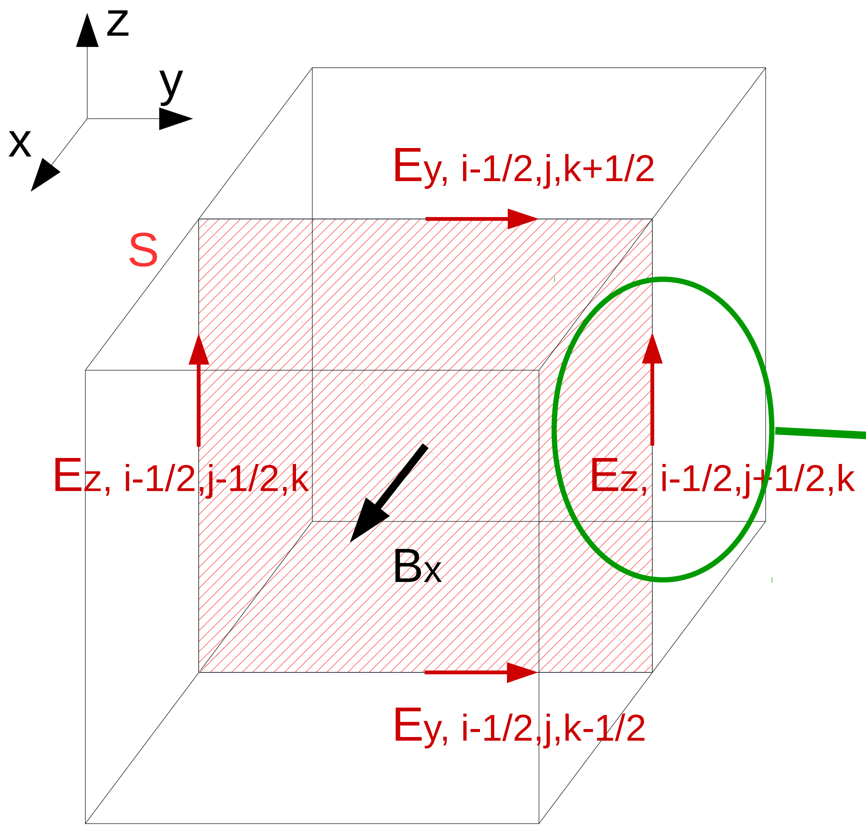
$$c_\mathrm{w} = \frac{\omega}{k} = \pm\frac{\eta_\mathrm{H} k}{2} + \sqrt{\left(\frac{\eta_\mathrm{H} k}{2}\right)^2 + c_\mathrm{A}^2}$$

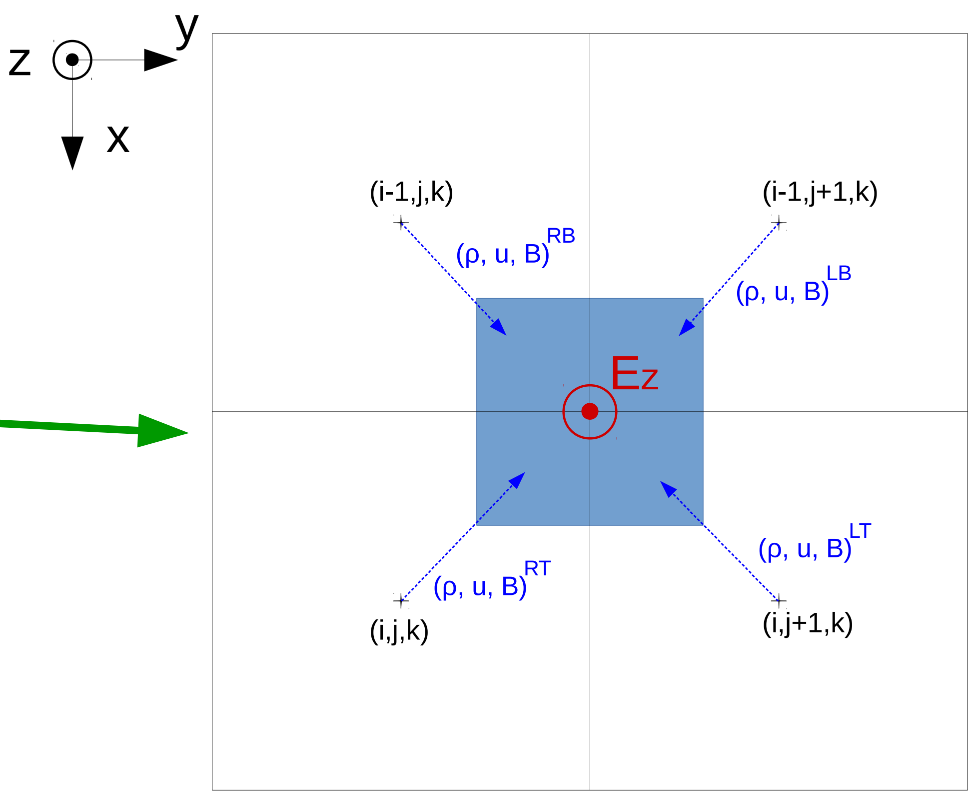
The Hall effect is dispersive :
- No transport of mass/energy
- *Whistler* waves

CONSTRAINED TRANSPORT

Ensures $\nabla \cdot B = 0$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E}$$





$$E_z^{RT} = u_x^{RT} B_y^{RT} - u_y^{RT} B_x^{RT}$$

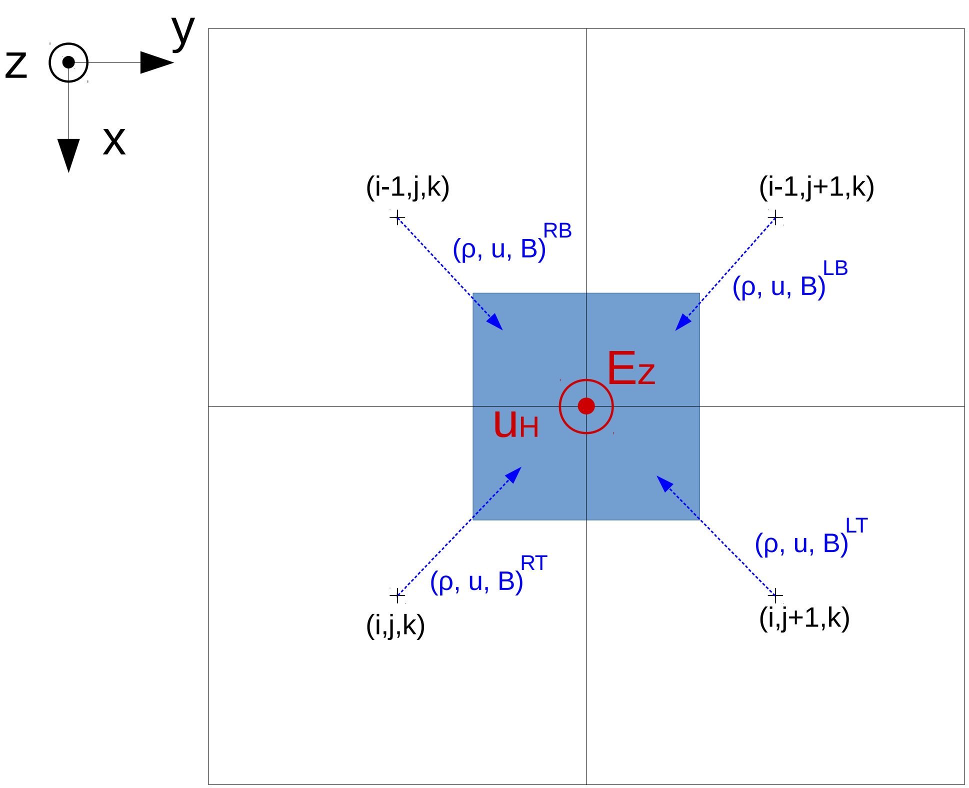
$$E_{z,i-1/2,j-1/2,k} = f\left(E_z^{LB}, E_z^{LT}, E_z^{RT}, E_z^{RB}, c_1, c_2, \ldots\right)$$

See *Teyssier+06* and *Fromang+06* for the implementation in RAMSES

HALL SPEED IMPLEMENTATION

Ideal MHD : $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\mathbf{u} \times \mathbf{B}] \longrightarrow E_z^{RT} = u_x^{RT} B_y^{RT} - u_y^{RT} B_x^{RT}$

Hall MHD : $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [(\mathbf{u} + \mathbf{u}_\mathrm{H}) \times \mathbf{B}] \longrightarrow E_z^{RT} = (u_x^{RT} + u_{\mathrm{H},x}) B_y^{RT} - (u_y^{RT} + u_{\mathrm{H},y}) B_x^{RT}$



$$\mathbf{u}_\mathrm{H} = \eta_\mathrm{H}(\overline{\rho}, \overline{T}, \overline{\zeta}, \overline{B}) \frac{\mathbf{J}}{\overline{B}}$$

$$c_\mathrm{w} = \frac{\eta_\mathrm{H}\pi}{2\Delta x} + \sqrt{\left(\frac{\eta_\mathrm{H}\pi}{2\Delta x}\right)^2 + c_\mathrm{A}^2}$$

New characteristic speed in HLL

CFL CONDITION

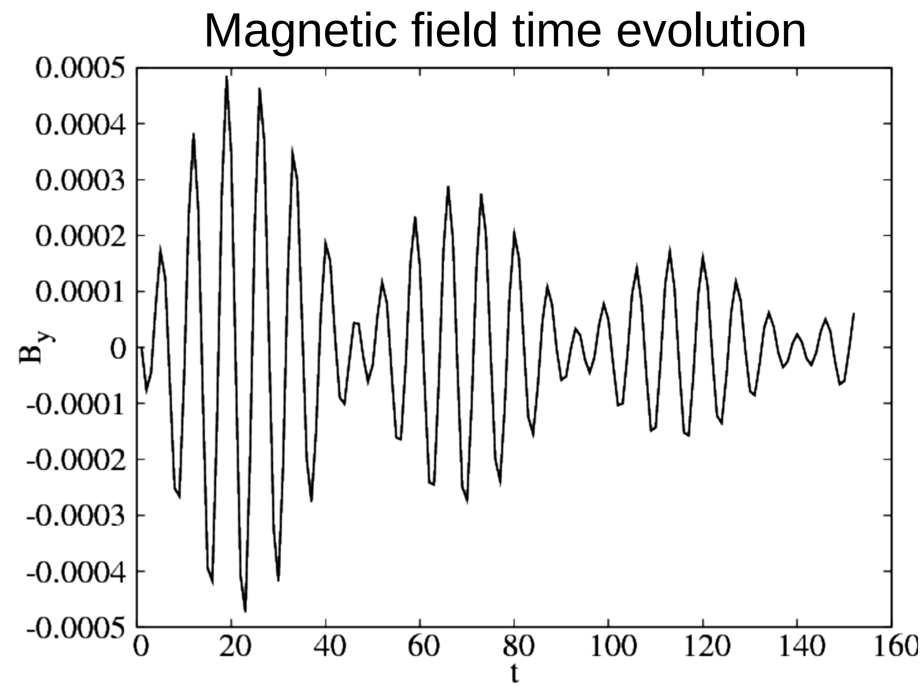
$$\Delta t_\mathrm{Hall} \approx \frac{\Delta x}{c_\mathrm{w}} \propto \Delta x^2$$

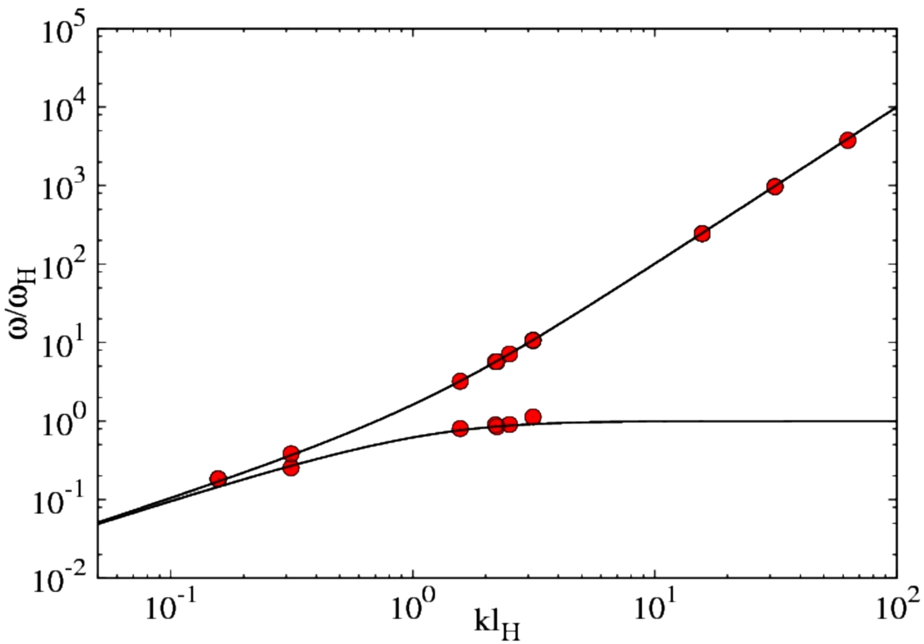
WHISTLER WAVE PROPAGATION

$$k \in \{5, 6, ...20\}$$

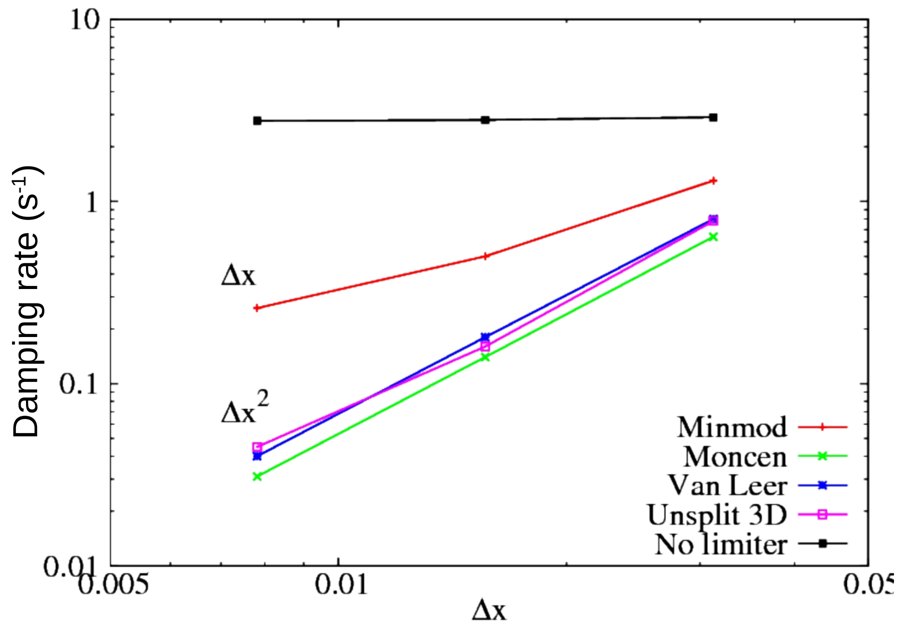
$$\eta_{\mathrm{H}} \in [10^{-3}, 1]$$

$$\Delta x \in \left[\frac{1}{128}, \frac{1}{32}\right]$$





Dispersion relation



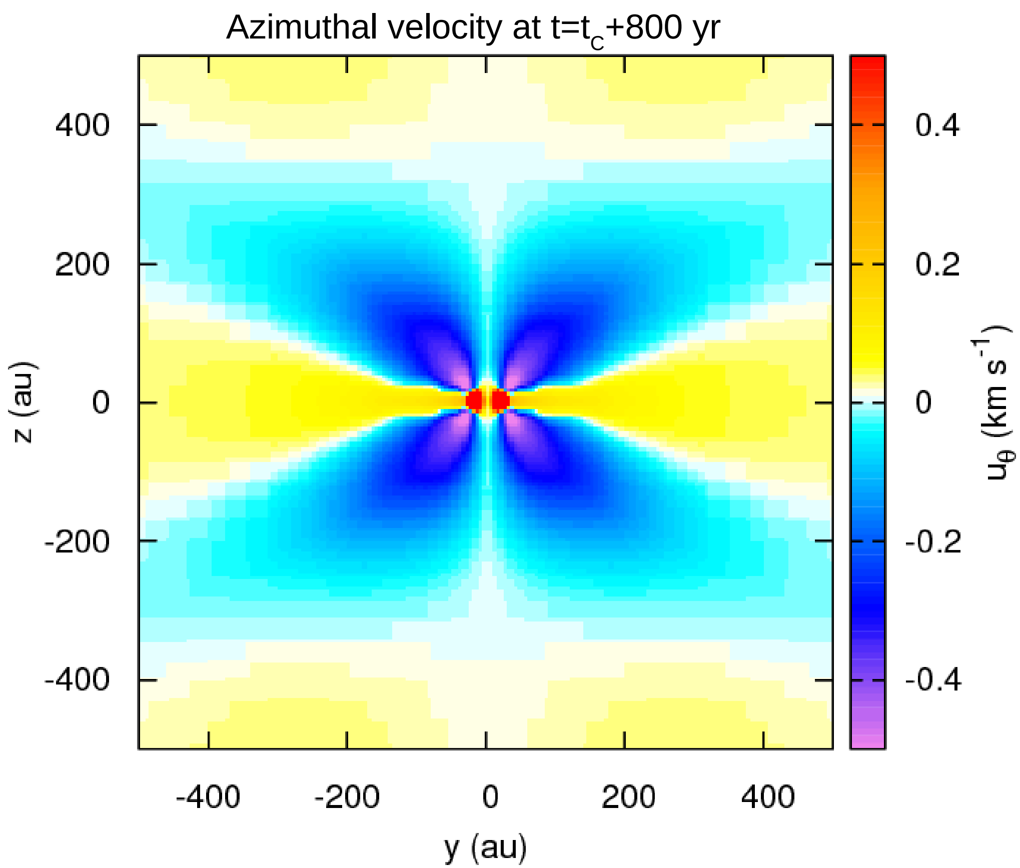
Convergence study

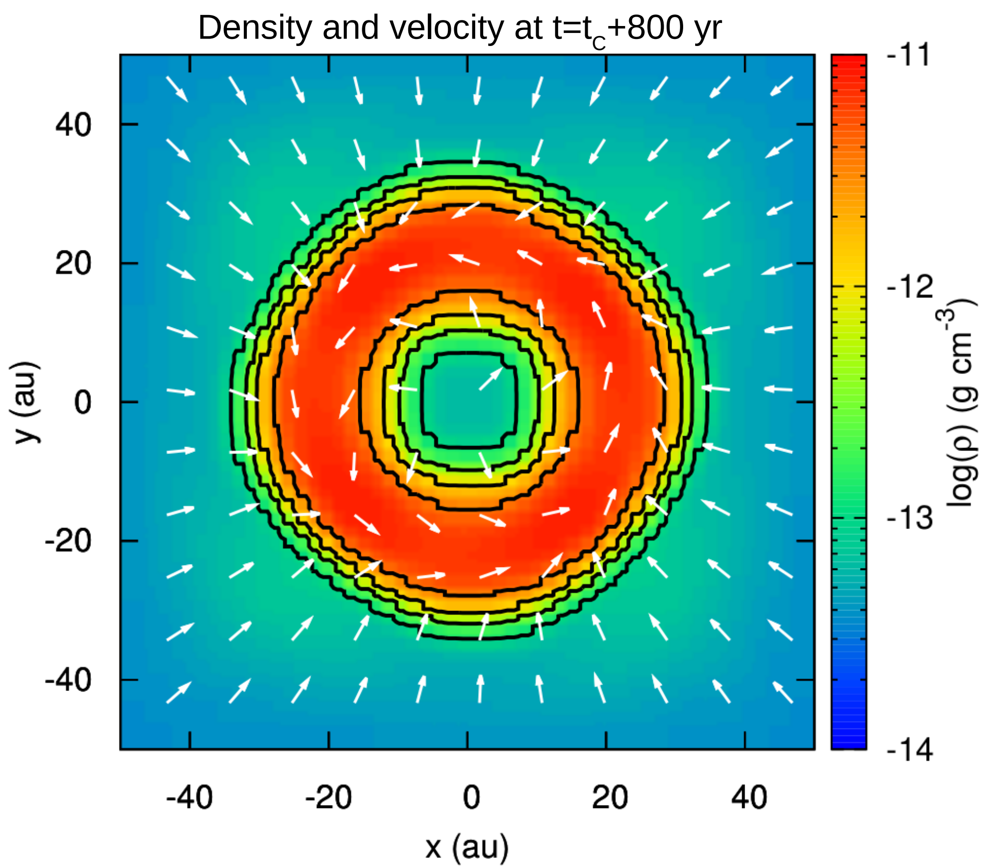
DENSE CORE COLLAPSE SIMULATIONS

- Uniform sphere $(M=1.5\,M_{sol}, \alpha=0.25, T=10\,K)$
- No rotation $(\beta=0)$
- Uniform magnetic field $(\mu=7)$
- Hall effect only $(\eta_H=10^{20}\,cm^2\,s^{-1})$

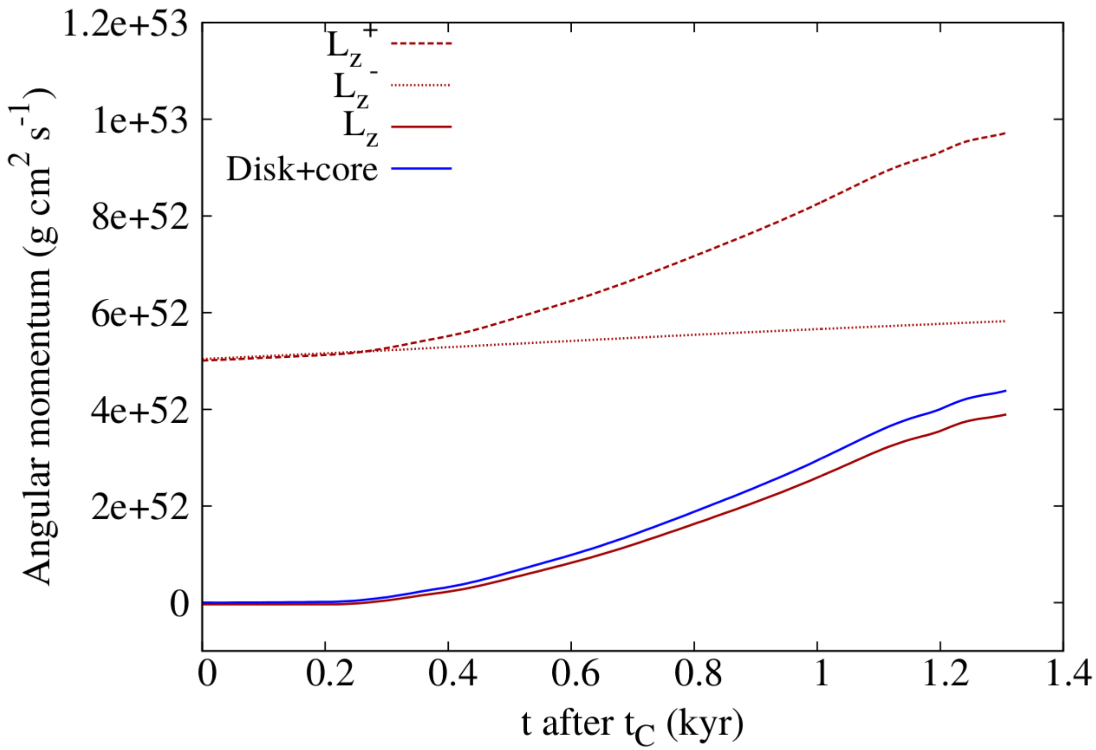
Rotation generated by the Hall effect
→ Disk
→ Counter-rotating envelopes

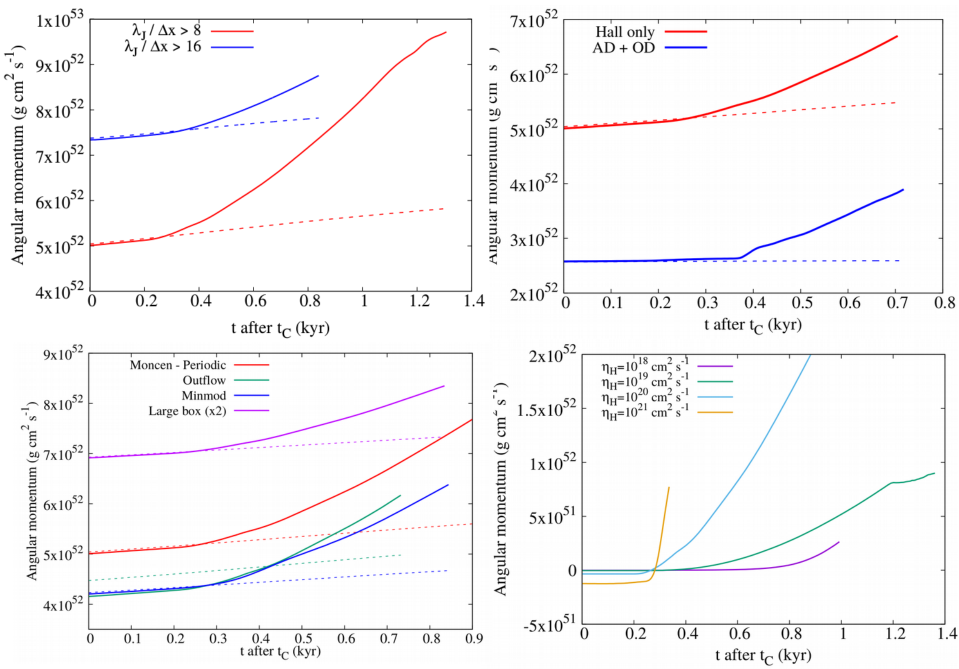
Tsukamoto+15,17, Wurster+16,17





THE ANGULAR MOMENTUM PROBLEM





Increase of the angular momentum from 300 yr after the formation of the first Larson core for every simulation with the Hall Effect, whatever

- The initial conditions
- The physics
- The numerical parameters

Dissipation of whistler waves ?

Numerical resolution ?

Accretion shock ?

Conclusions

THE HALL EFFECT :
- Dispersive term
- May play a significant role in star formation

IMPLEMENTATION :
- Modification of the CT scheme and 2D-Riemann solver
- Dispersion relation OK, Second order convergence

SIMULATIONS :
- We reproduce the characteristics features of the HE
- The angular momentum is not conserved

OUTLOOK :
- Compare with other codes (PHANTOM, ATHENA++...)
- Run more tests

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