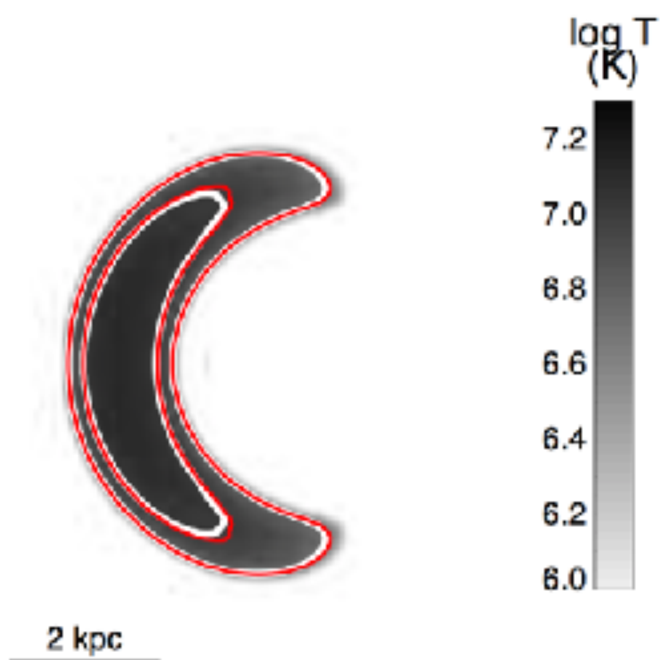
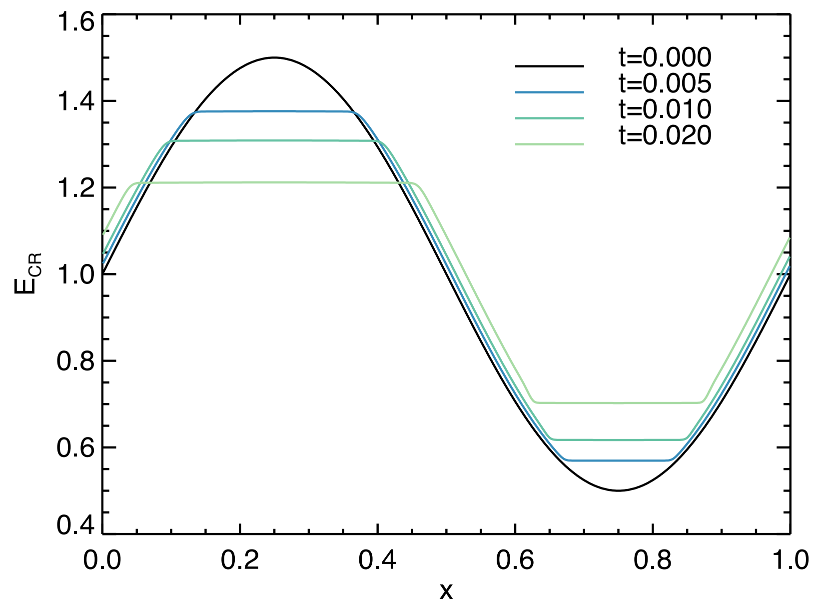
Cosmic ray physics in Ramses

Anisotropic Diffusion

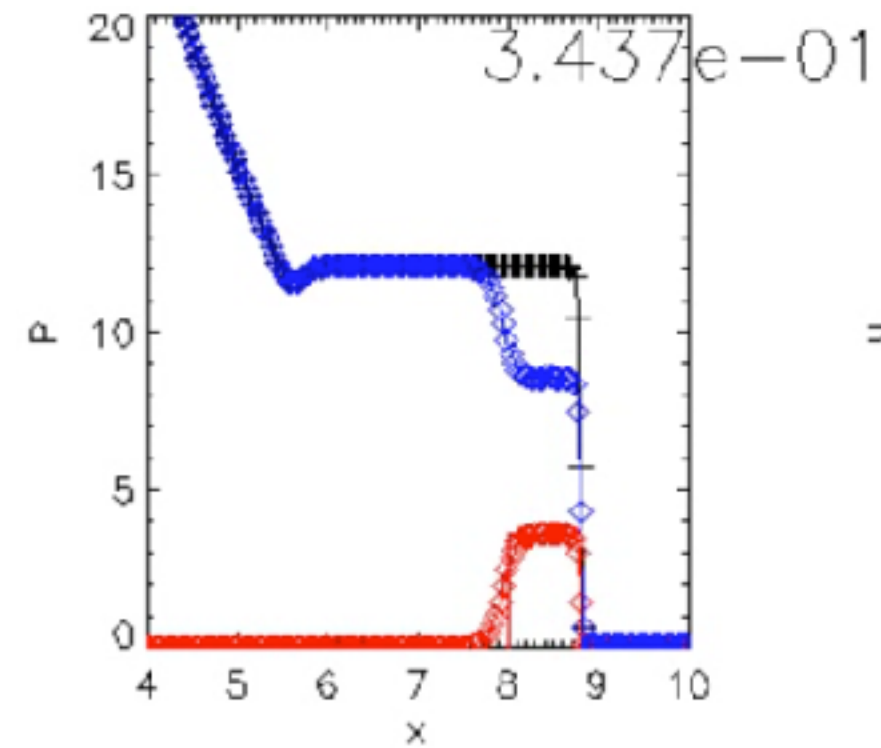


Dubois & Commerçon 16

Streaming



Shock acceleration



Dubois+ in prep.

Yohan Dubois

Loann Brahimi, Benoît Commerçon, Gohar Dashyan, Alexandre Marcowith

Why should I care about cosmic rays?

- <u>Equipartition of energies</u> (kinetic ~ thermal ~ magnetic ~ cosmic rays) in galaxy formation problems: intra-cluster medium, active galactic nuclei jets, galactic winds, interstellar medium

- As a relativistic population of particles <u>their adiabat and losses are different from that of the gas</u>

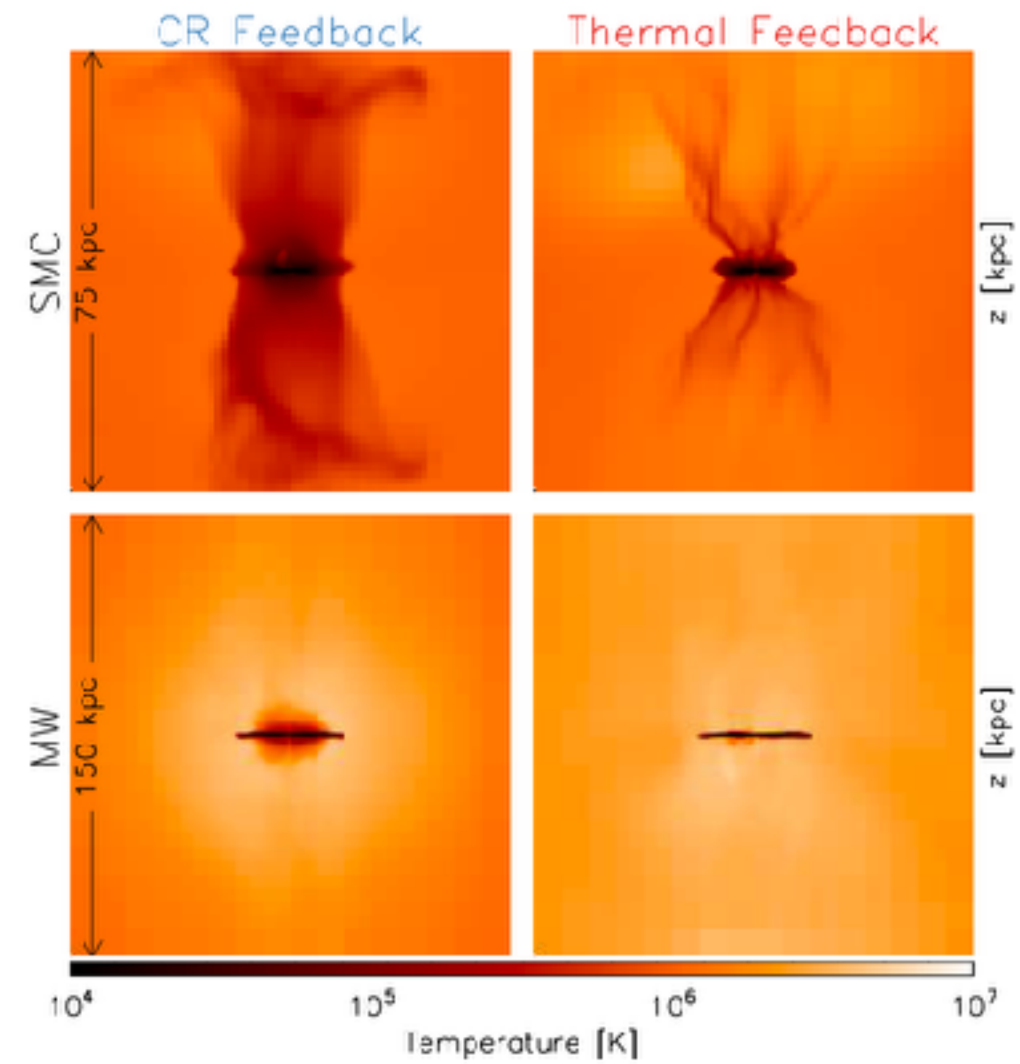
- <u>Diffusion</u> is a key aspect of cosmic ray transport

- Cosmic rays are <u>generated at shocks</u>:
  supernovae, jets, cosmic infall

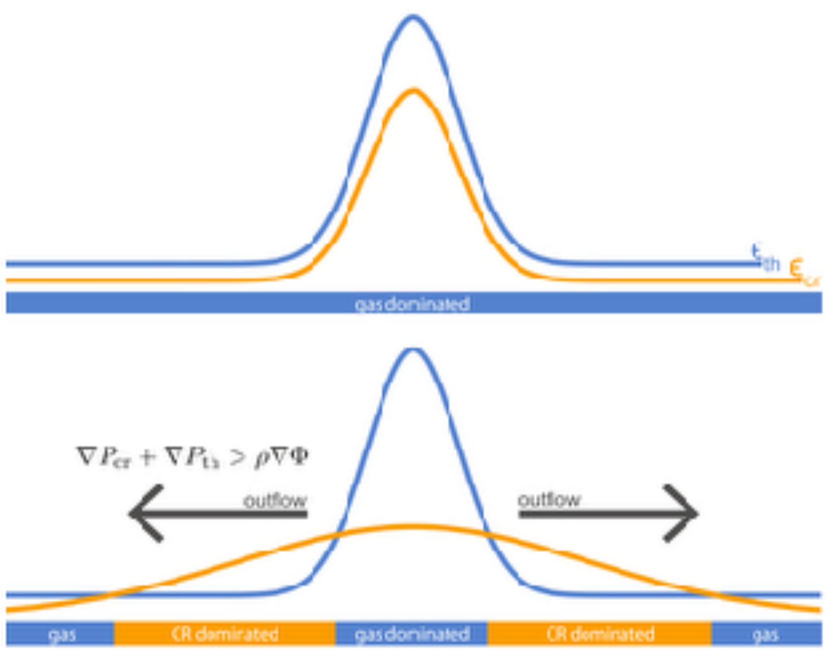
Why we should care
Galactic scale

- CRs reinforce the strength of galactic winds as they diffuse into low-gas densities.

- Amplified galactic dynamos. (Hanasz et al, 2004)



Booth et al (2013)



Salem & Bryan (2014)

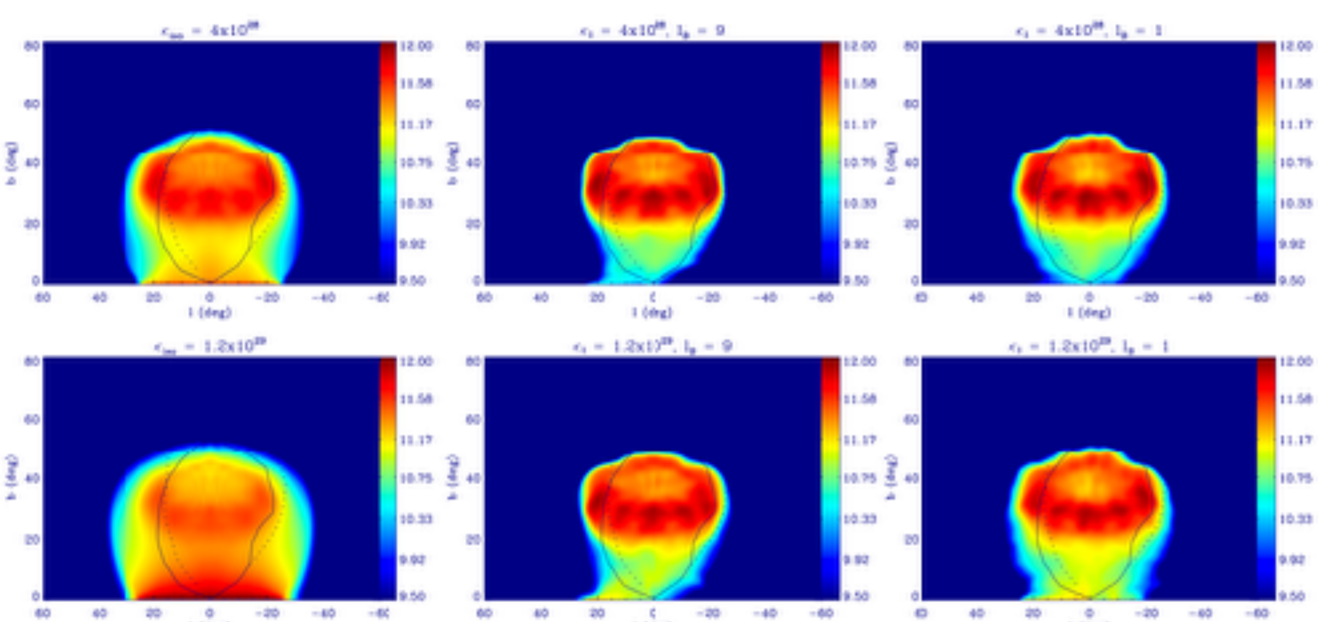
See also: Jubelgas et al (2008),
Uhlig et al (2012),
Hanasz et al (2013)

Why we should care

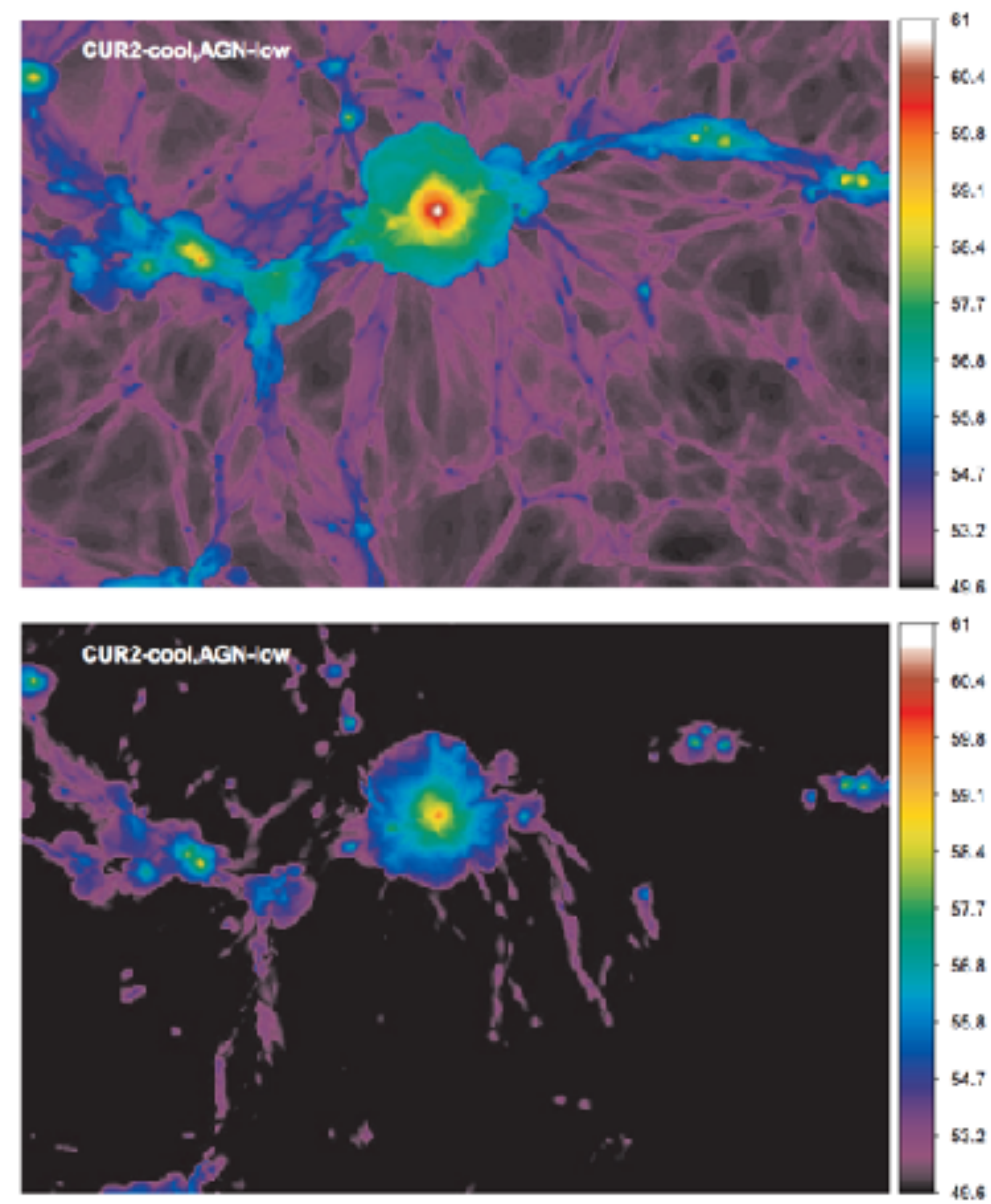
Cluster scale

- In galaxy clusters, cosmic gas produce strong shocks and accelerate particles: radio relics

- AGN jets are filled with CRs. Energy deposit on larger scales than the jet scale?



Yang+ 12



Vazza+ 16

Conservation laws for cosmic ray (ideal) magneto-hydrodynamics and non local thermodynamical equilibrium

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \boldsymbol{u}) = 0 \,, \quad (1)$$

mass

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla.\left(\rho \boldsymbol{u}\boldsymbol{u} + P_{\rm tot} - \frac{\boldsymbol{B}\boldsymbol{B}}{4\pi}\right) = 0 \,, \quad (2)$$

momentum

$$\frac{\partial e}{\partial t} + \nabla.\left((e + P_{\rm tot})\boldsymbol{u} - \frac{\boldsymbol{B}(\boldsymbol{B}.\boldsymbol{u})}{4\pi}\right) = -\nabla.\boldsymbol{F}_{\bf cond} \,, \quad (3)$$

total energy

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) = 0 \,, \quad (4)$$

magnetic field

$$\frac{\partial e_{\rm E}}{\partial t} + \nabla.(e_{\rm E}\boldsymbol{u}) = -P_{\rm E}\nabla.\boldsymbol{u} - \nabla.\boldsymbol{F}_{\rm cond,E} + \mathcal{H}_{\rm EI} \quad (5)$$

electron energy (non LTE)

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left(e_{\rm CR}\boldsymbol{u} + (e_{\rm CR} + P_{\rm CR})\boldsymbol{u}_{\rm st}\right) = - P_{\rm CR}\nabla.\boldsymbol{u} - \nabla.\boldsymbol{F}_{\rm CR} + \mathcal{L}_{\rm st} + \mathcal{H}_{\rm acc} + \mathcal{L}_{\rm loss} \quad (6)$$

CR energy

CR evolution

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left(e_{\rm CR}\vec{u} + (e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = -P_{\rm CR}\nabla.\vec{u} - \nabla.\vec{F}_{\rm CR} + \mathcal{L}_{\rm st} + \mathcal{H}_{\rm acc} + \mathcal{L}_{\rm loss}$$

Shock acceleration ($\mathcal{H}_{\rm acc}$)

Advection ($e_{\rm CR}\vec{u}$)

Streaming ($(e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}$)

Work ($-P_{\rm CR}\nabla.\vec{u}$)

Diffusion ($\nabla.\vec{F}_{\rm CR}$)

Streaming heating of the gas ($\mathcal{L}_{\rm st}$)

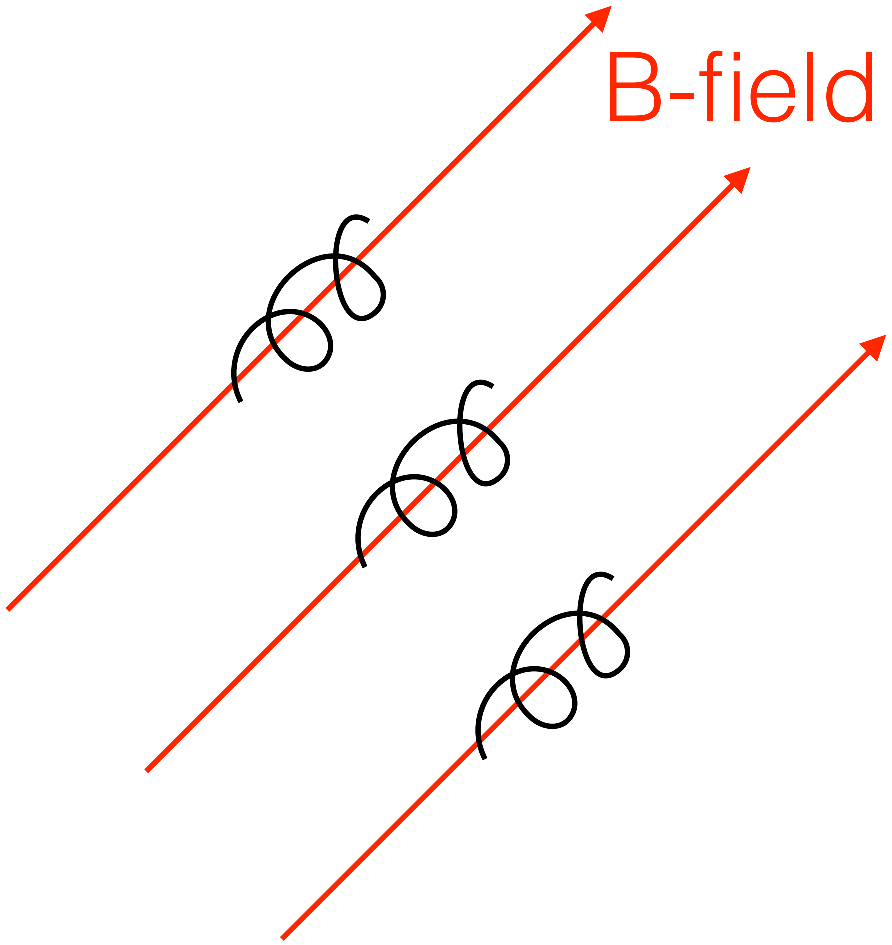
Radiative losses ($\mathcal{L}_{\rm loss}$)

$$\mathcal{L}_{\rm loss} = -7.5\times10^{-16} n_{\rm e} e_{\rm CR}\,{\rm erg\,s^{-1}\,cm^{-3}}$$

Let’s ignore everything but energy transport with convective advection and diffusion

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left(e_{\rm CR}\vec{u}\right) = -P_{\rm CR}\nabla.\vec{u} - \nabla.\vec{F}_{\rm CR}$$

Diffusion is anisotropic



Anisotropy:
Particles are charged and hence follow magnetic field lines as they gyrate around

Diffusion:
Kinetic description of CR propagation by a Fokker-Planck equation (*Schlickeiser 2002*):
Pitch angle scattering
<=> pitch angle diffusion term
<=> diffusion in position

$$\frac{\partial e_{\rm CR}}{\partial t} = -\nabla.\vec{F}_{\rm CR} = -\nabla.\left(-D_{||}\vec{b}(\vec{b}.\nabla e_{\rm CR})\right) - \nabla.\left(-D_{\rm iso}\nabla e_{\rm CR}\right)$$

parallel component — isotropic component

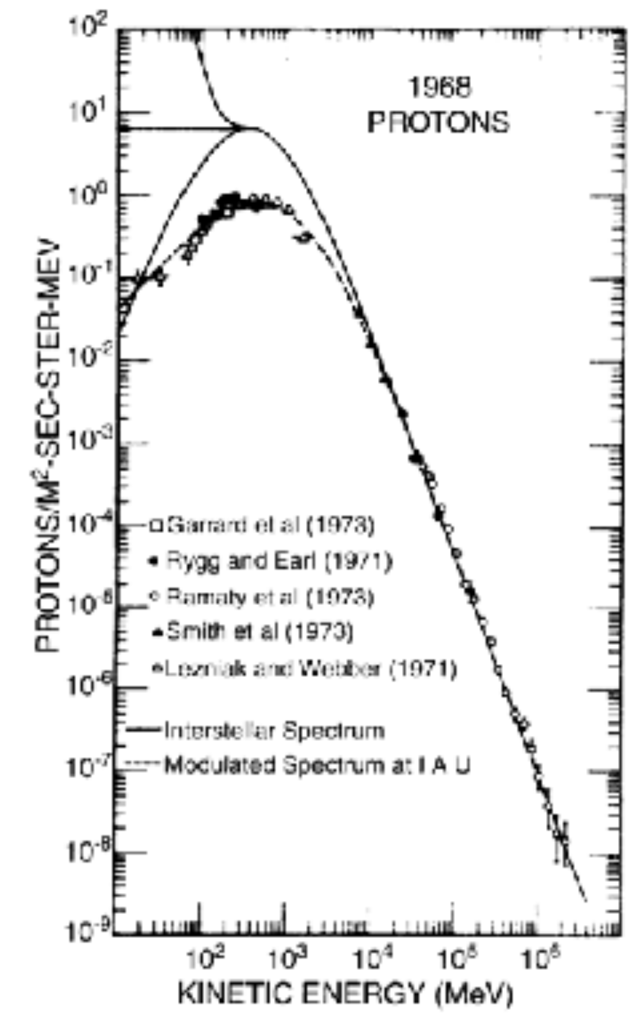
$$D_{||} = D_0 - D_{\rm iso}$$

A note of caution

The diffusion coefficient depends on the shape of turbulence in Fourier space (e.g. Kolmogorov versus Kraichnan) and the energy/momentum (p) distribution of CRs.

$$D = D_0(p/p_0)^s \qquad s = 0.3 - 0.7$$

The spatial CR momentum distribution will vary with acceleration and ageing of CRs: diffusion, shock acceleration and radiative losses depend on momentum



Fulks 75

We greatly idealize a complex problem by treating CRs with a single energy bin

Take a given distribution of CRs in momentum space
=> an effective diffusion coefficient
=> an effective adiabatic index
=> an effective radiative loss terms

Anisotropic diffusion in Ramses

Dubois & Commerçon 16

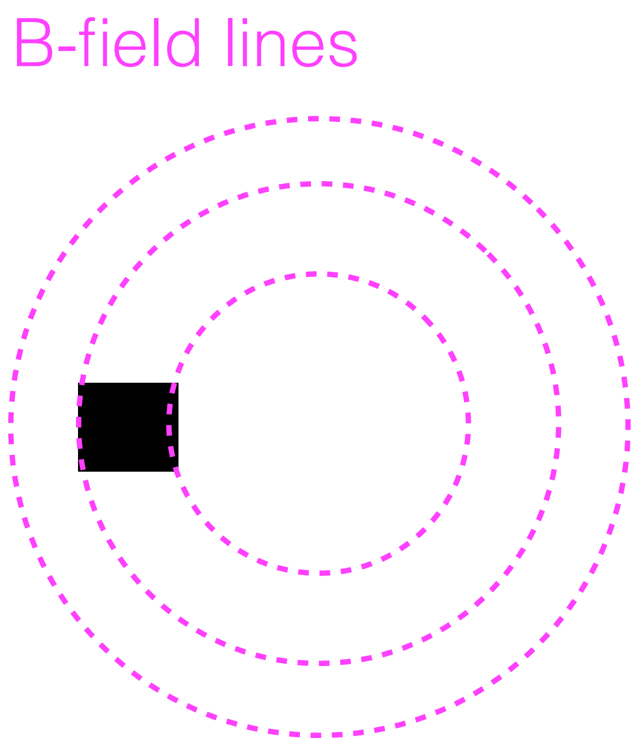
- Fully implicit solver (faster than explicit due to stringent timestep constraint $dt=dx^2/(2D)$)
- Fully adaptive solver (exploit the adaptive mesh and adaptive time step hierarchy)
- Works for both CR diffusion and thermal conduction
- Can do anisotropic diffusion with arbitrary degree of anisotropy (e.g. isotropic)
- Only works with mhd=.true.

Tests

- 1D step function w/ AC
- 1D cosmological shock w/ 2 Temp and AC
- <u>2D AC in a B-loop</u>
- Sovinec test for numerical diffusion
- <u>Supernova explosion in 3D w/ MHD + 2CTAC + CRAD*</u>

Dubois & Commerçon 16

2D AC in a B-loop



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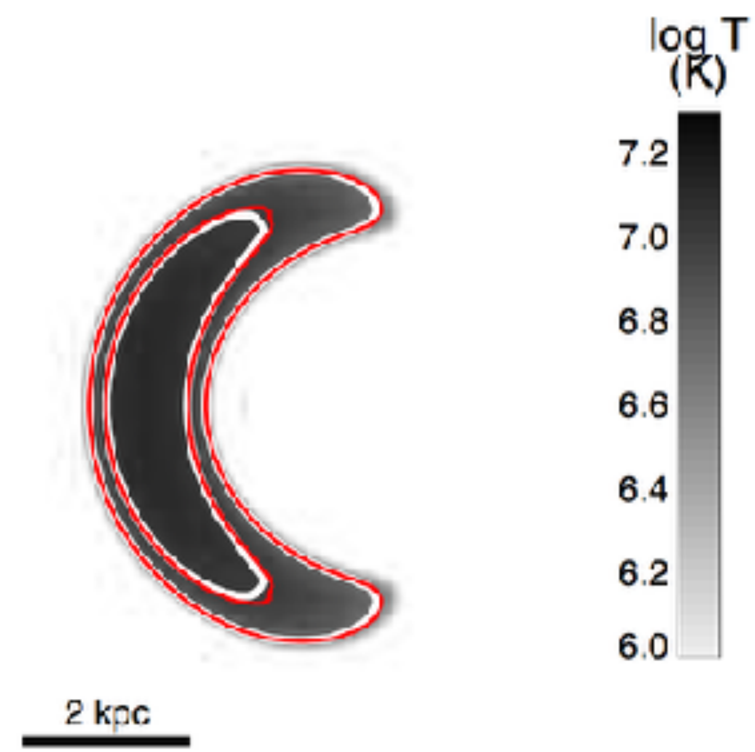
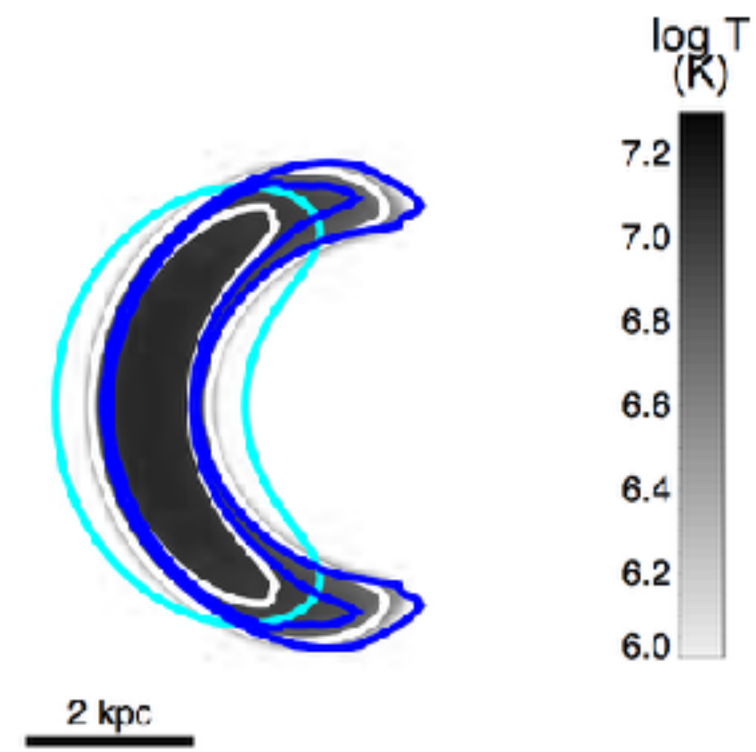
2D AC in a B-loop

Testing the AMR



with 5 AMR levels
uniform grid

Testing the isotropic component



Kiso=0.001
Kiso=0.01
Kiso=0.1

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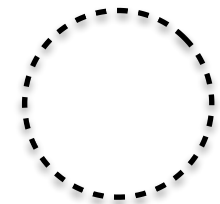
Supernova explosion in 3D

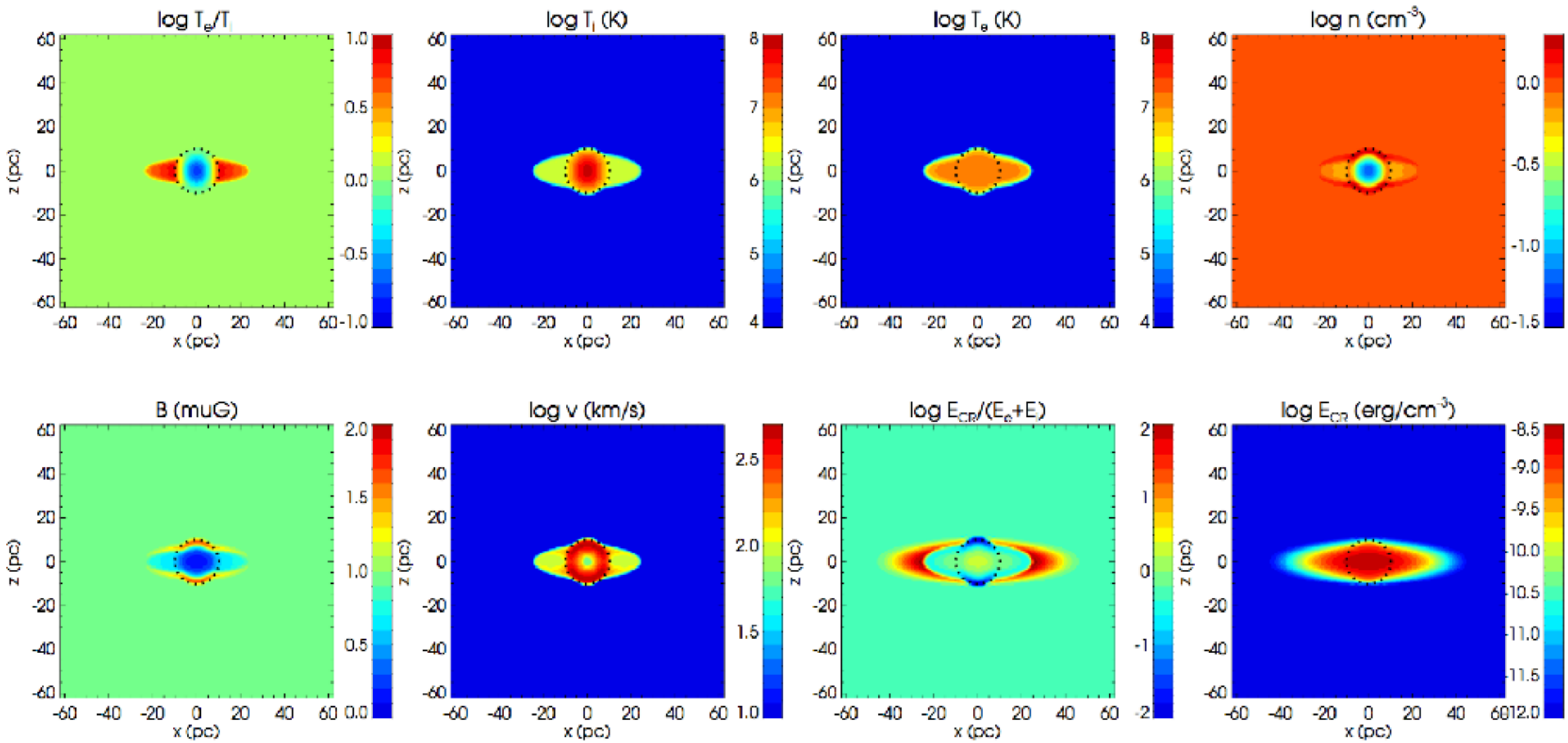
10^{51} erg in 1 cm^{-3}

1/3 in E_E, 1/3 in E_I, 1/3 in E_{CR}



Sedov front position

<u>after 5 kyr</u>



initial B-field

Dubois & Commerçon 16

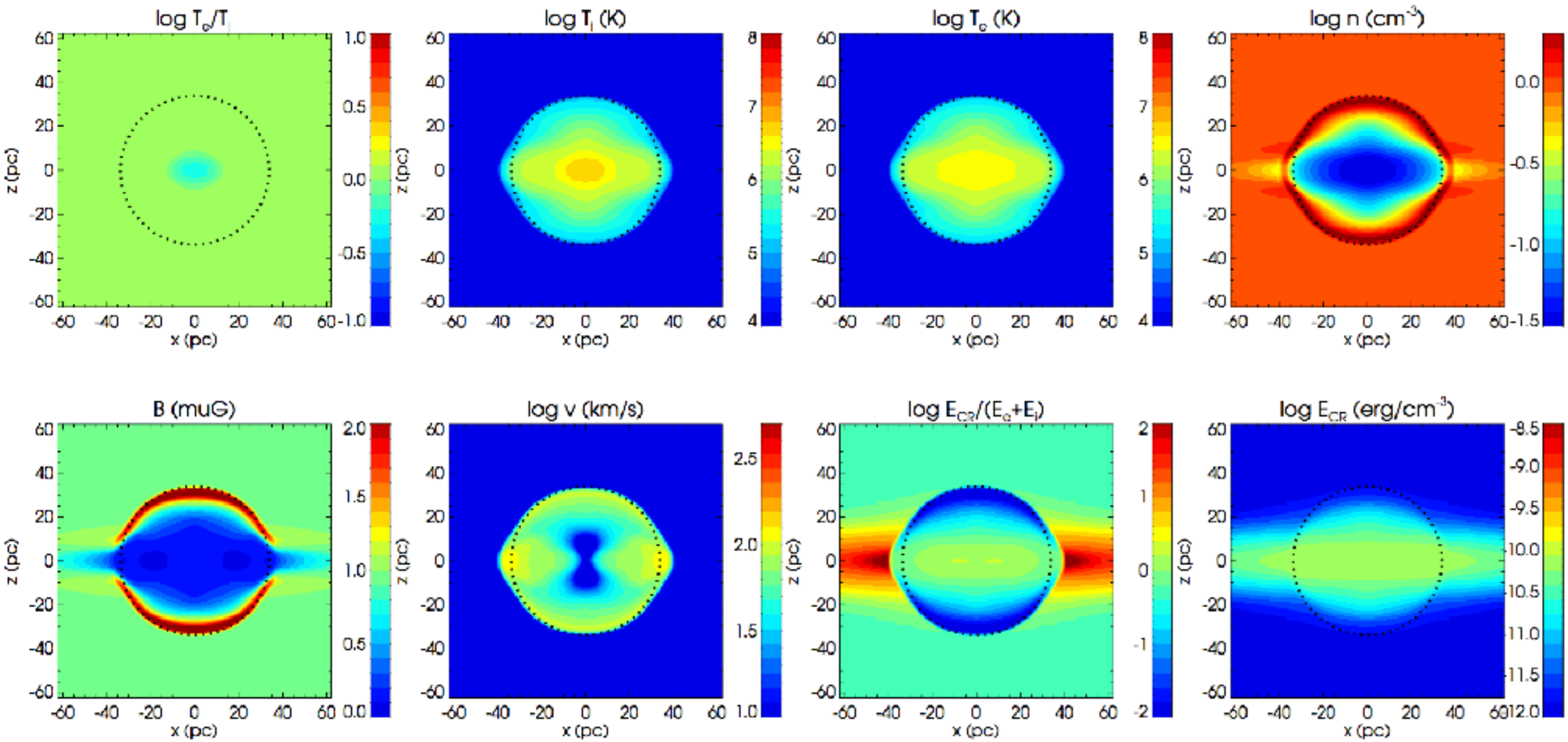
Supernova explosion in 3D

10^{51} erg in 1 cm^{-3}

1/3 in E_E, 1/3 in E_I, 1/3 in E_{CR}

Sedov front position

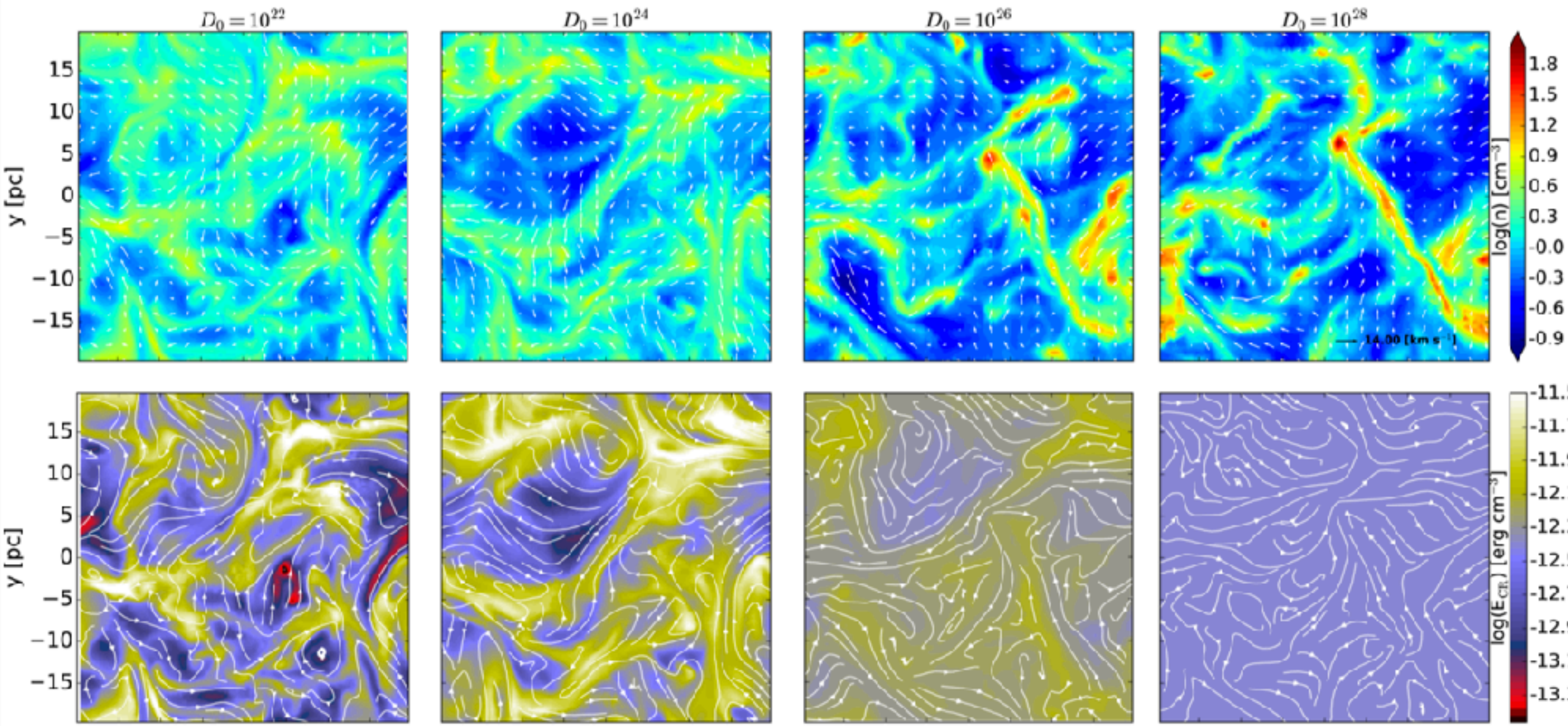
after 100 kyr



initial B-field

Dubois & Commerçon 16

CRs and ISM gas distribution



Commerçon, Marcowith & Dubois, sub.

CR streaming

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left(e_{\rm CR}\vec{u} + (e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = -P_{\rm CR}\nabla.\vec{u} - \nabla.\vec{F}_{\rm CR} + \mathcal{L}_{\rm st} + \mathcal{H}_{\rm acc} + \mathcal{L}_{\rm loss}$$

Advection

Streaming

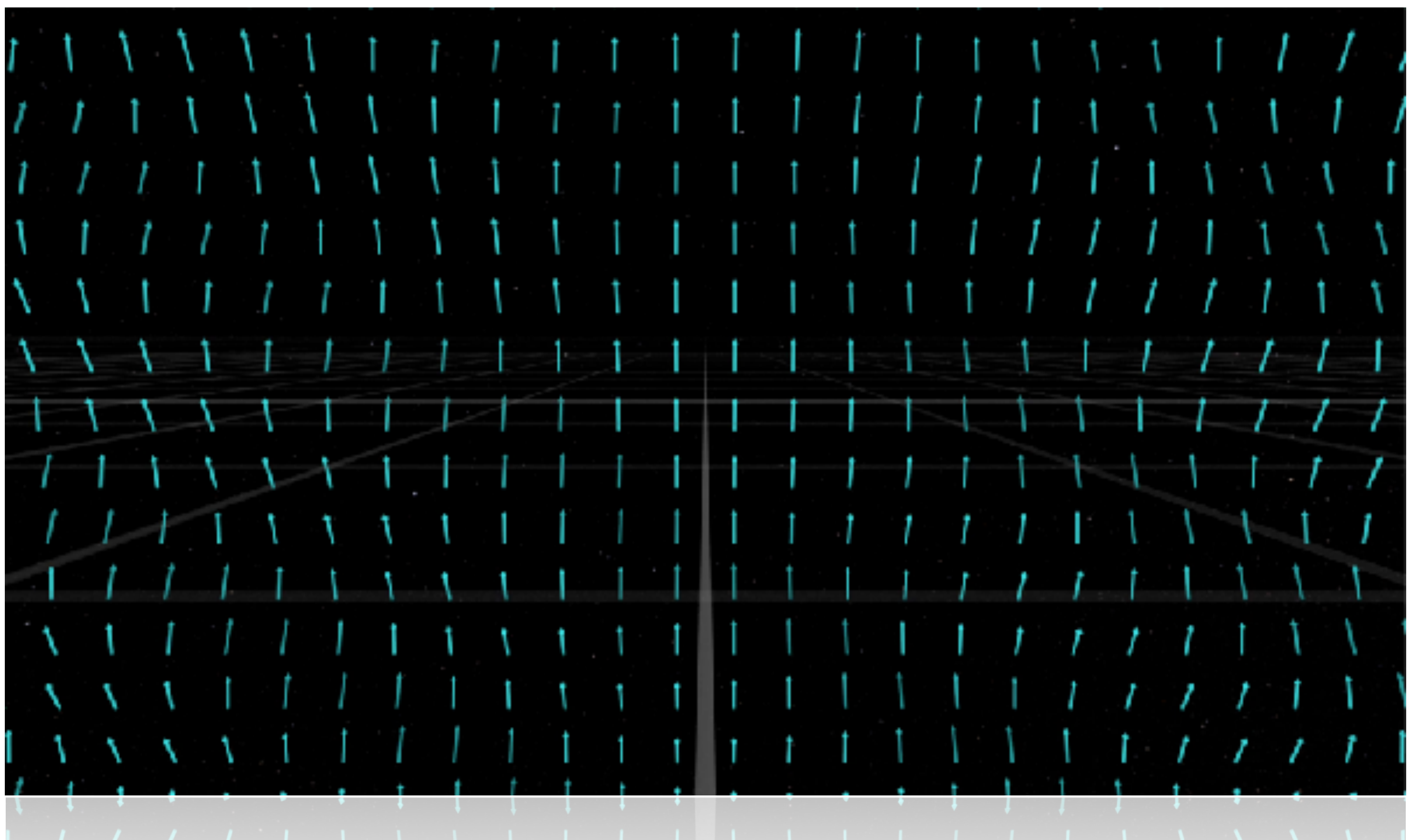
Work

Diffusion

Streaming heating of the gas

Shock acceleration

Radiative losses



CRs as they stream along B excite Alfvén waves, which perturb B

Subsequently, CRs gyrate along field lines and feel the perturbation of the B field

-> they stream down their own gradient at the Alfven velocity and heat the gas

$$\vec{u}_{\rm st} = -\vec{u}_{\rm A}{\rm sign}(\vec{b}.\nabla e_{\rm CR})$$

Visualisation by Tom Bridgman for NASA

CR streaming

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left(e_{\rm CR}\vec{u} + (e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = -P_{\rm CR}\nabla.\vec{u} - \nabla.\vec{F}_{\rm CR} + \mathcal{L}_{\rm st} + \mathcal{H}_{\rm acc} + \mathcal{L}_{\rm loss}$$

Shock acceleration

Advection

Streaming

Work

Diffusion

Radiative losses

Streaming heating of the gas

Let's ignore everything but the streaming terms

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left((e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = \vec{u}_{\rm st}.\nabla P_{\rm CR} = \mathcal{L}_{\rm st}$$

Streaming heating is always a loss/gain term for the CR/thermal component

$$\vec{u}_{\rm st} = -\vec{u}_{\rm A}{\rm sign}(\vec{b}.\nabla e_{\rm CR})$$

$$\mathcal{L}_{\rm st} = -{\rm sign}(\vec{b}.\nabla e_{\rm CR})\vec{u}_{\rm A}.\nabla P_{\rm CR} = -\frac{B}{\sqrt{4\pi\rho}}(\gamma-1)\frac{\left(\vec{b}.\nabla e_{\rm CR}\right)^2}{|\vec{b}.\nabla e_{\rm CR}|}$$

CR streaming

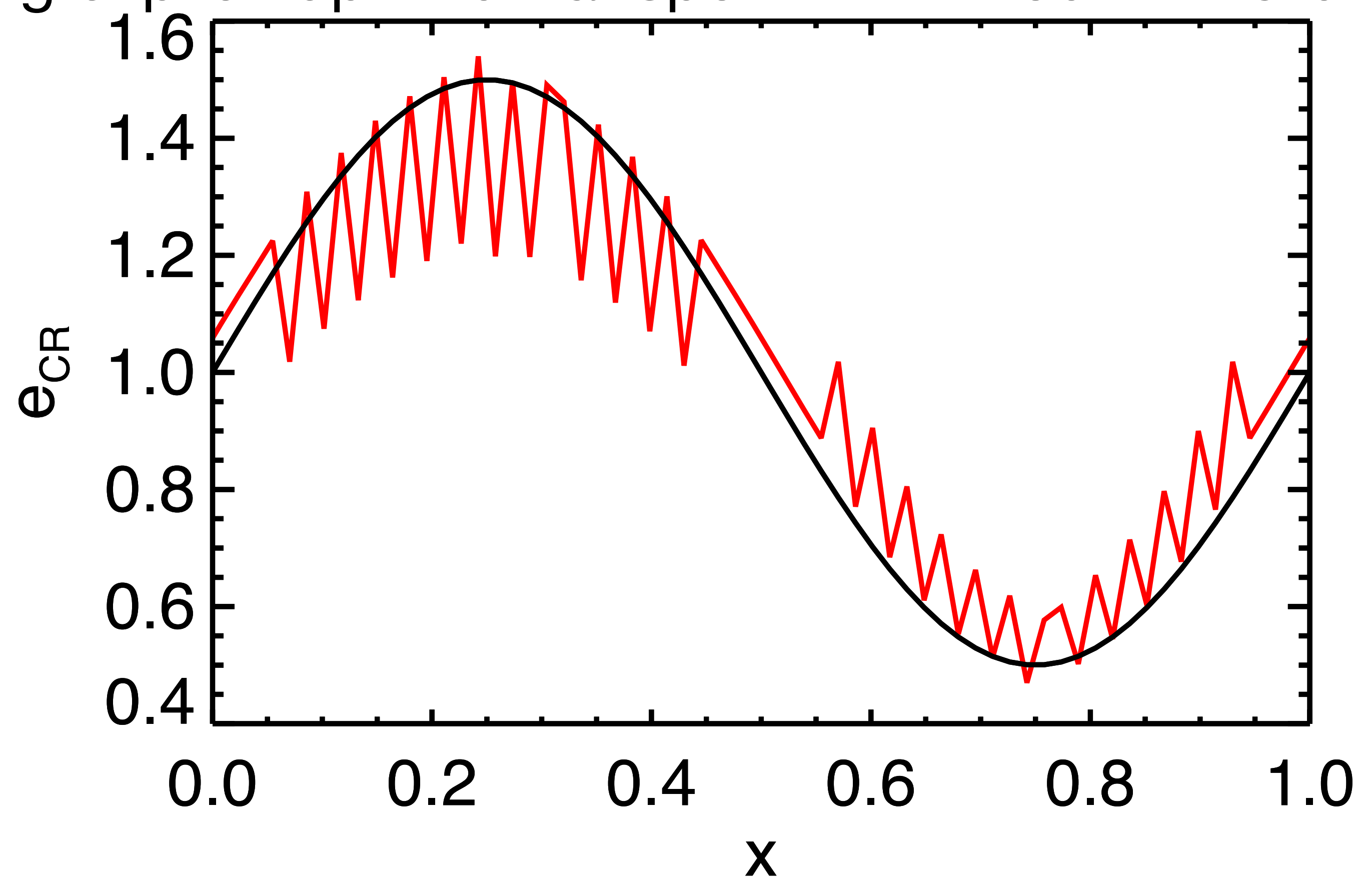
$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left((e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = \mathcal{L}_{\rm st}$$

Sounds like a genuine advection term, right?

$$\vec{u}_{\rm st} = -\vec{u}_{\rm A}{\rm sign}(\vec{b}.\nabla e_{\rm CR})$$

with uniform u_A=1 and n_x=64

using explicit upwind transport + MinMod TVD & dt=dx/|u_A|



CR streaming

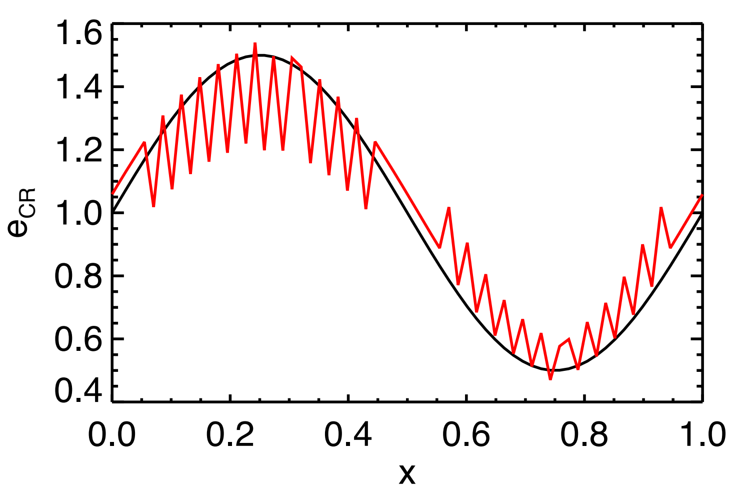
$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left((e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = \mathcal{L}_{\rm st}$$

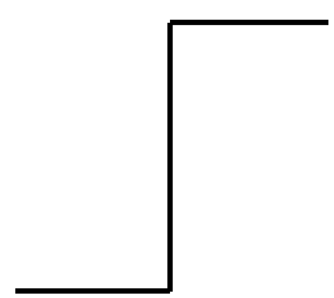
Sounds like a genuine advection term, right?

$$\vec{u}_{\rm st} = -\vec{u}_{\rm A}\,{\rm sign}(\vec{b}.\nabla e_{\rm CR})$$

The actual timestep for a stable explicit method solving the streaming term is

$$\Delta t = \frac{\nabla^2 e_{\rm CR}}{4 u_{\rm A} e_{\rm CR}} \Delta x^3$$

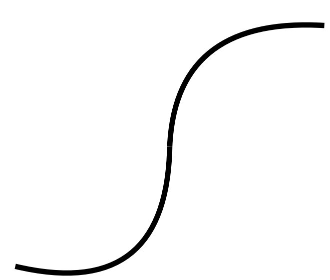




Approximate

$${\rm sign}(\vec{b}.\nabla e_{\rm CR})$$

with



$$\tanh(\vec{b}.\nabla e_{\rm CR}/\epsilon)$$

$$\epsilon = \text{a few } \Delta x$$

but

$$\Delta t \propto \Delta x^2$$

Sharma, Colella & Martin 09

CR streaming

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left((e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = \mathcal{L}_{\rm st}$$

Sounds like a genuine advection term, right?

$$\vec{u}_{\rm st} = -\vec{u}_{\rm A}{\rm sign}(\vec{b}.\nabla e_{\rm CR})$$

$$\nabla.\left((e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = \nabla.\left(-\frac{(e_{\rm CR} + P_{\rm CR})|B|}{|\vec{b}.\nabla e_{\rm CR}|\sqrt{4\pi\rho}}\vec{b}(\vec{b}.\nabla e_{\rm CR})\right)$$

$$= \nabla.(-D_{\rm st}\vec{b}(\vec{b}.\nabla e_{\rm CR}))$$

It's a diffusion term with coefficient with complex dependencies
Let's use our implicit solver to ignore the $dt=dx^2/(2D)$ constraint!

it's mathematically equivalent (no approximation)

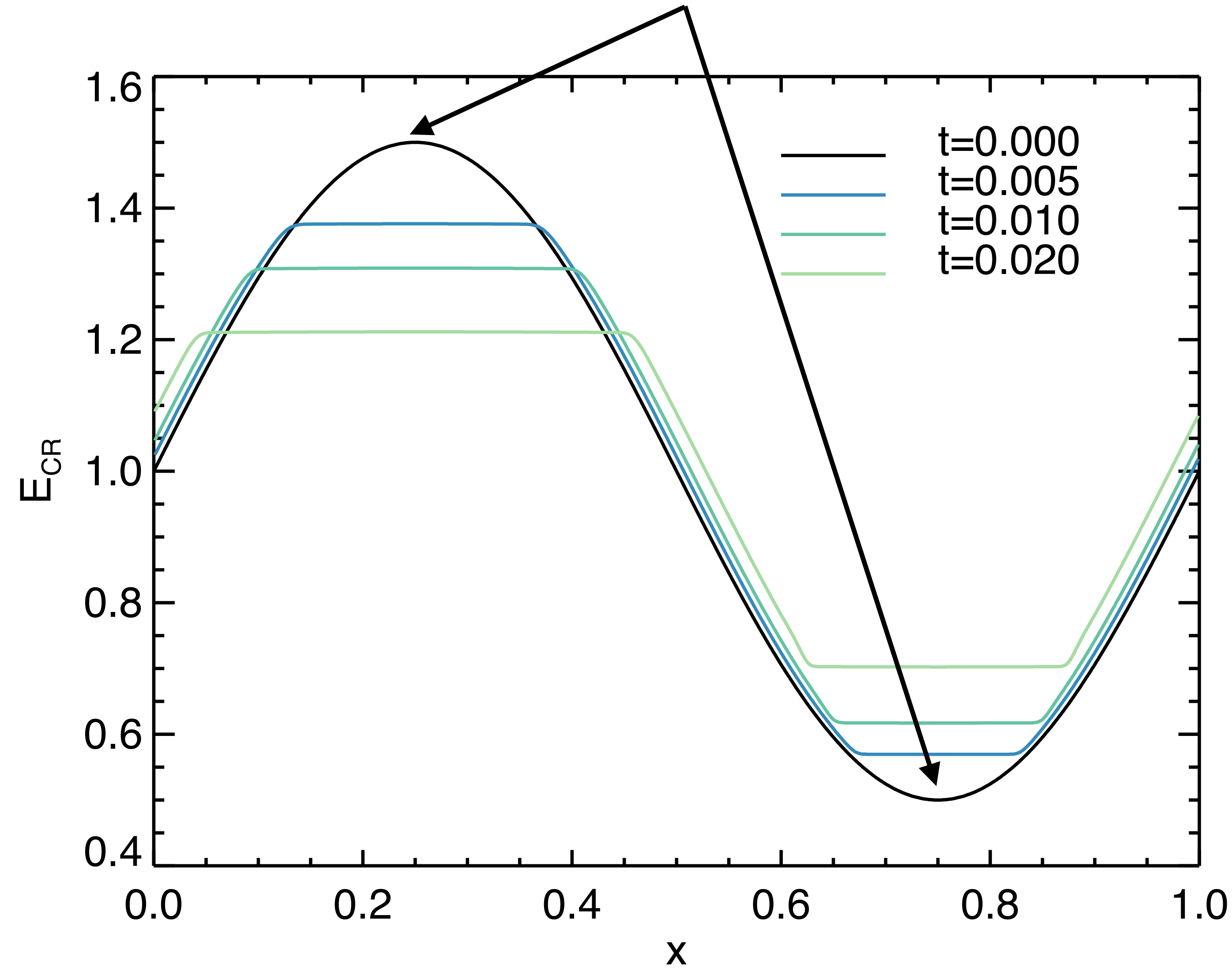
CR streaming

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla . ((e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}) = \mathcal{L}_{\rm st}$$

$$\vec{u}_{\rm st} = -\vec{u}_{\rm A}{\rm sign}(\vec{b}.\nabla e_{\rm CR})$$

$$D_{\rm st} = \frac{(e_{\rm CR} + P_{\rm CR})|B|}{|\vec{b}.\nabla e_{\rm CR}|\sqrt{4\pi\rho}}$$

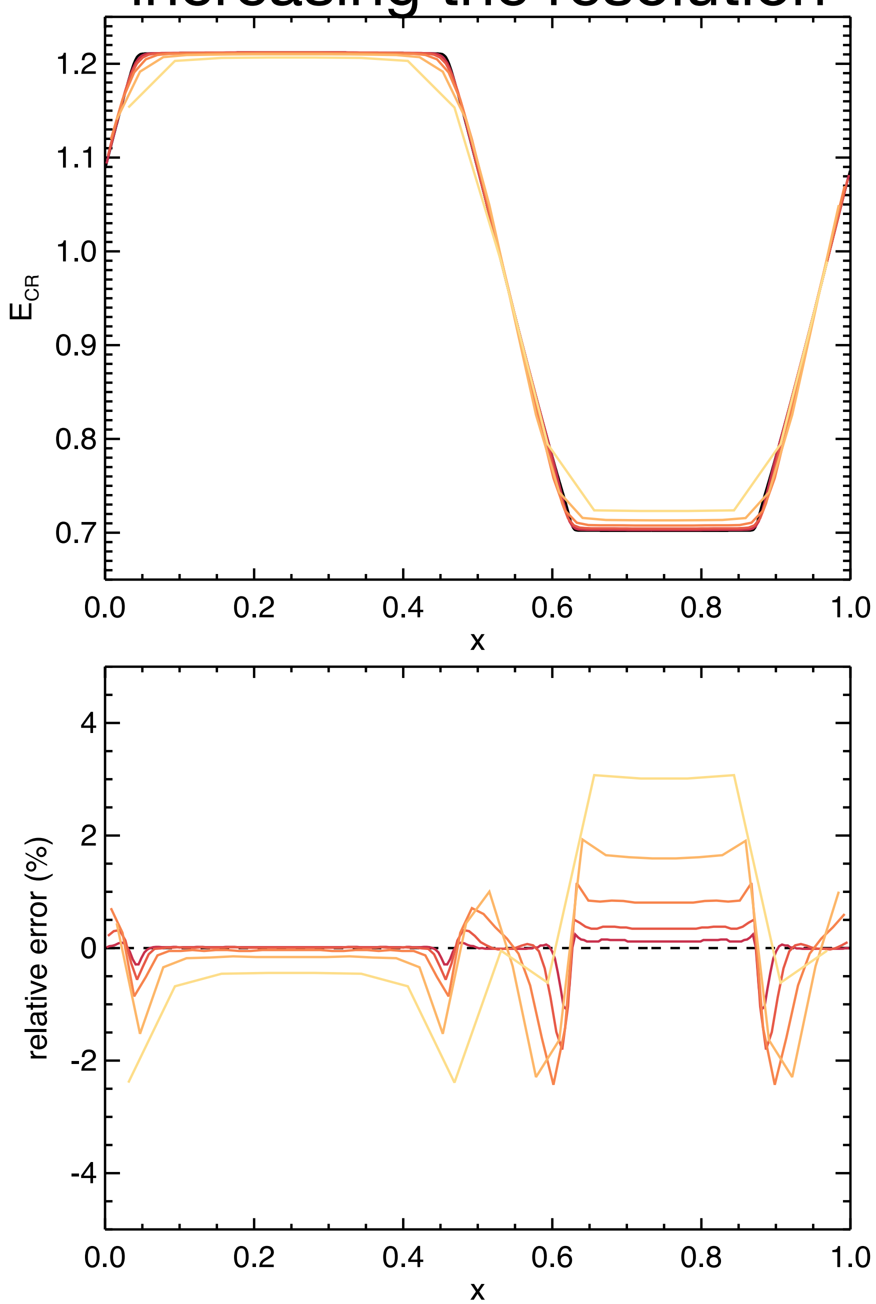
$$D_{\rm st} \rightarrow \infty \; (\nabla e_{\rm CR} \rightarrow 0)$$



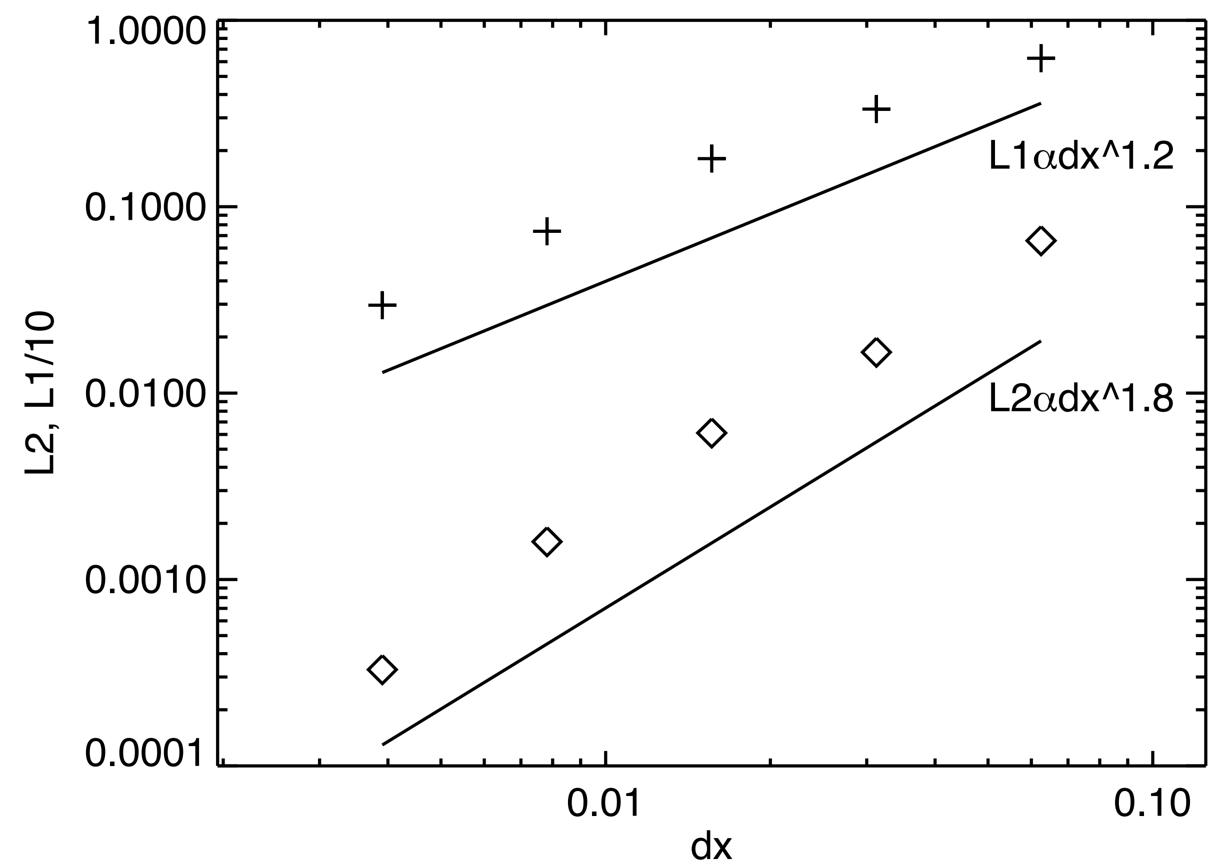
CR streaming

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left((e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = \mathcal{L}_{\rm st}$$

increasing the resolution



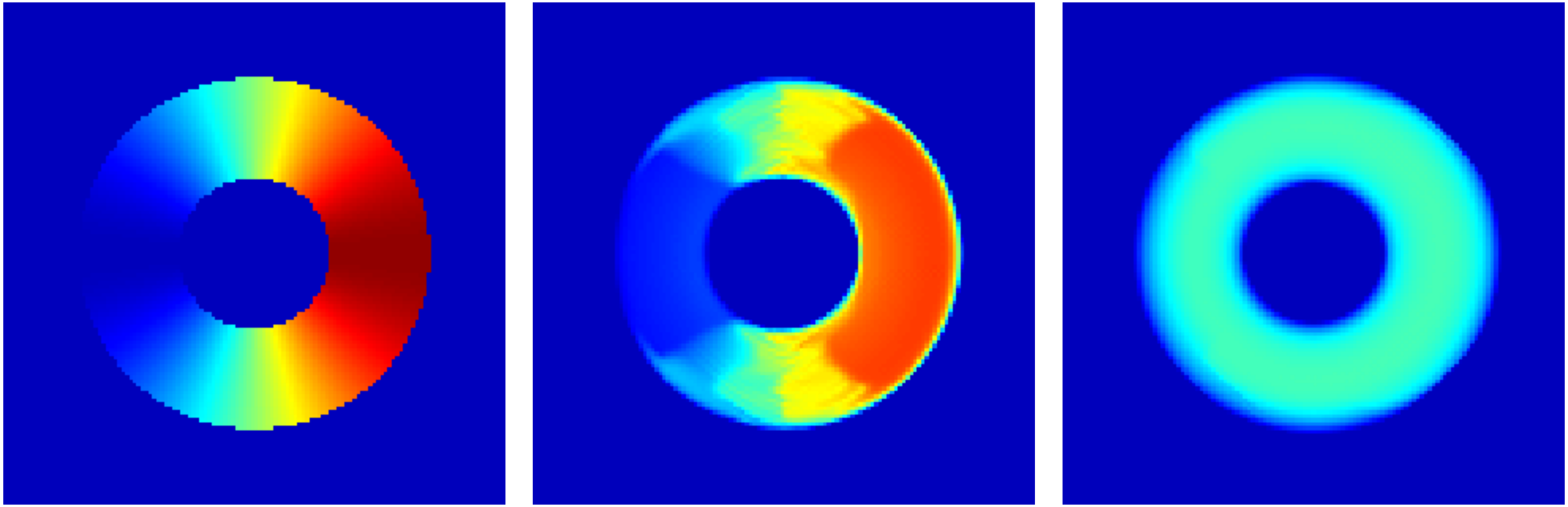
testing the numerical convergence



CR streaming

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla . ((e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}) = \mathcal{L}_{\rm st}$$

sinusoid in angle



CR shock acceleration

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left(e_{\rm CR}\vec{u} + (e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = -P_{\rm CR}\nabla.\vec{u} - \nabla.\vec{F}_{\rm CR} + \mathcal{L}_{\rm st} + \mathcal{H}_{\rm acc} + \mathcal{L}_{\rm loss}$$

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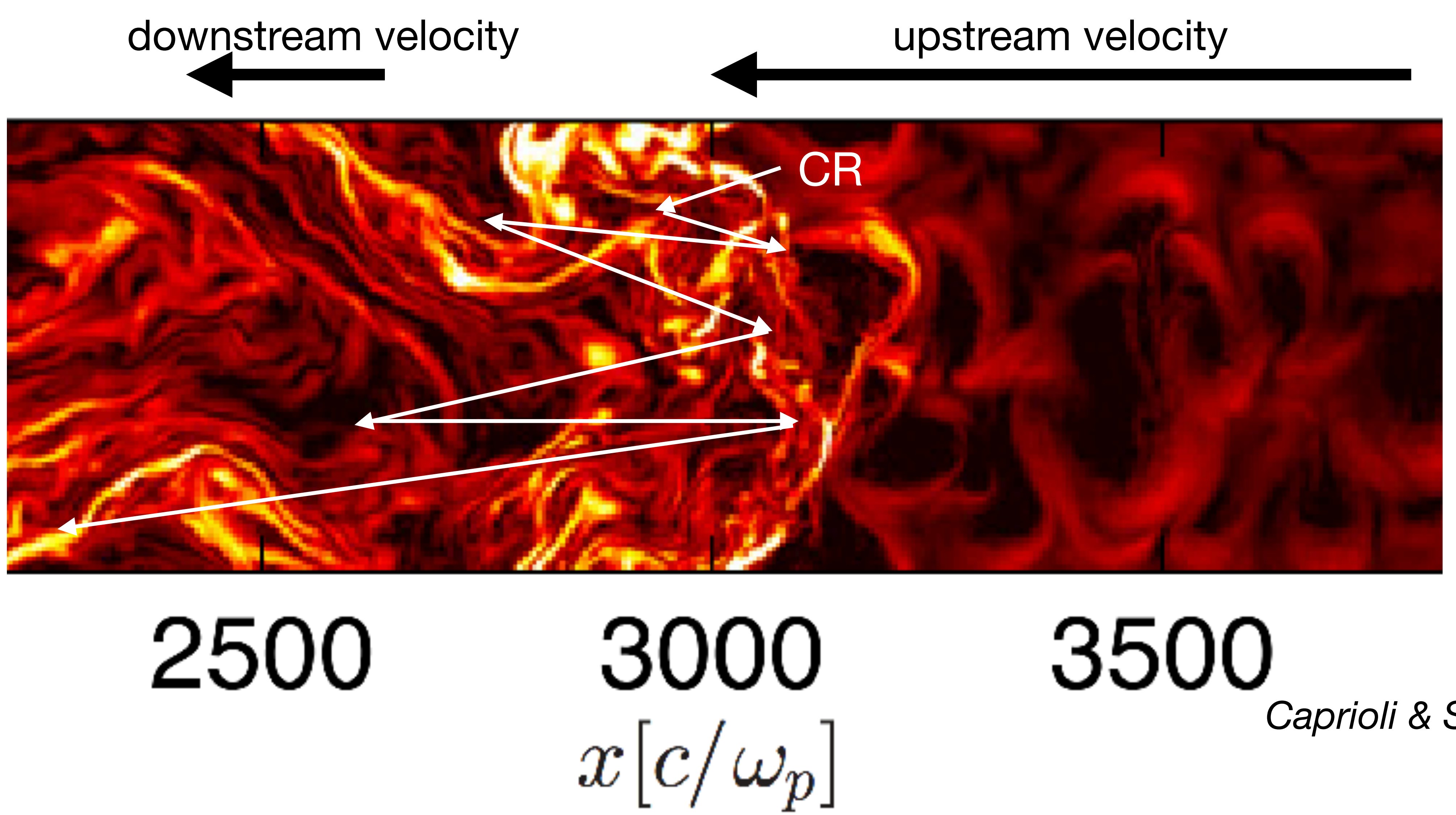

Caprioli & Spitkovsky 14

CR shock acceleration

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla.\left(e_{\rm CR}\vec{u} + (e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = -P_{\rm CR}\nabla.\vec{u} - \nabla.\vec{F}_{\rm CR} + \mathcal{L}_{\rm st} + \mathcal{H}_{\rm acc} + \mathcal{L}_{\rm loss}$$

Shock acceleration

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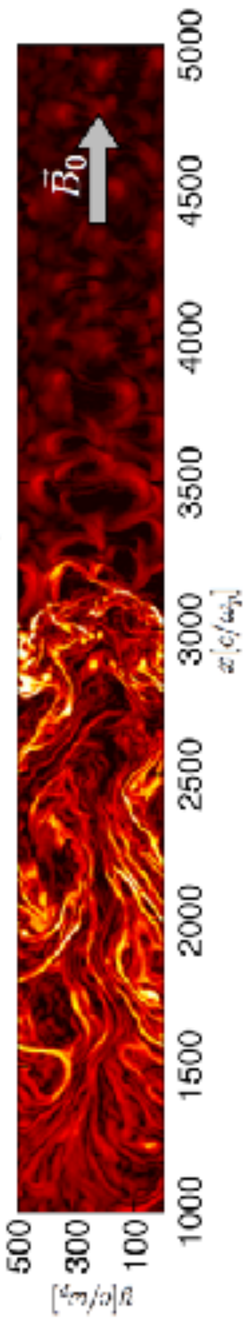
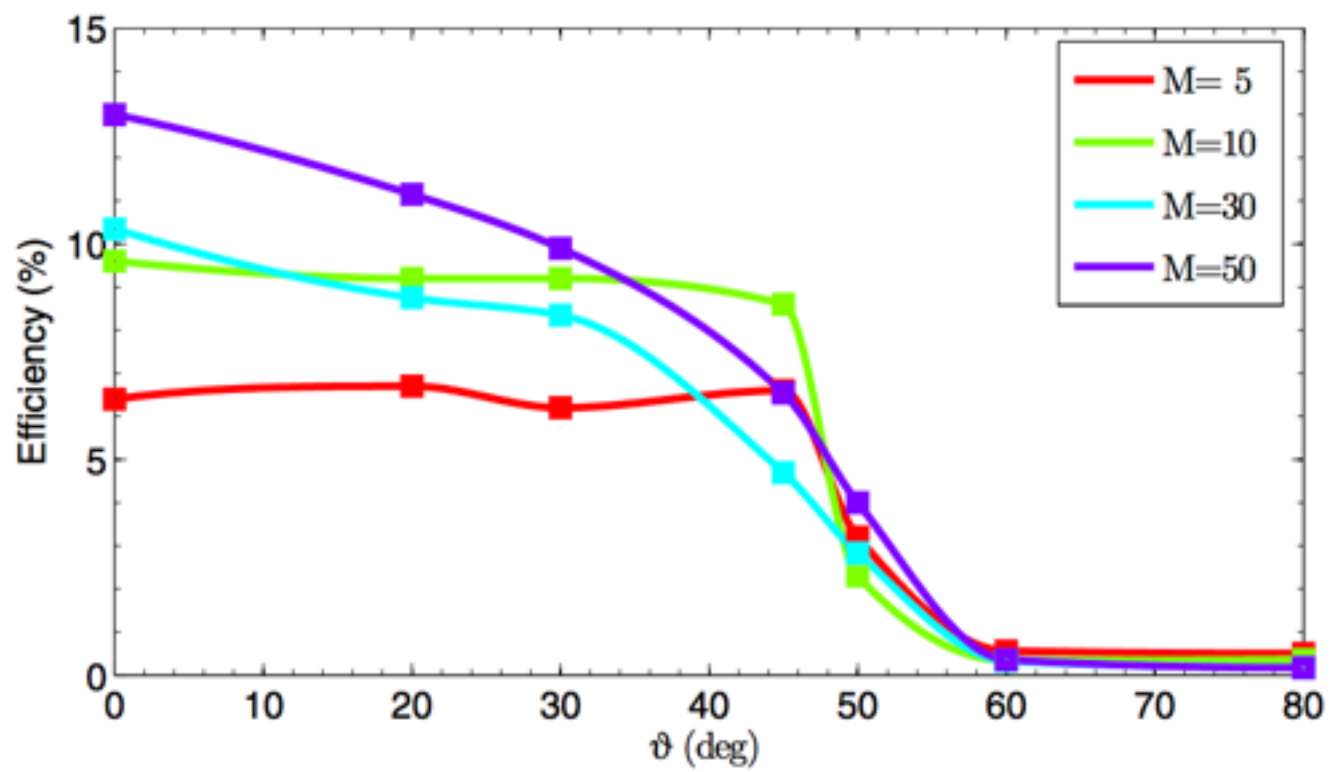
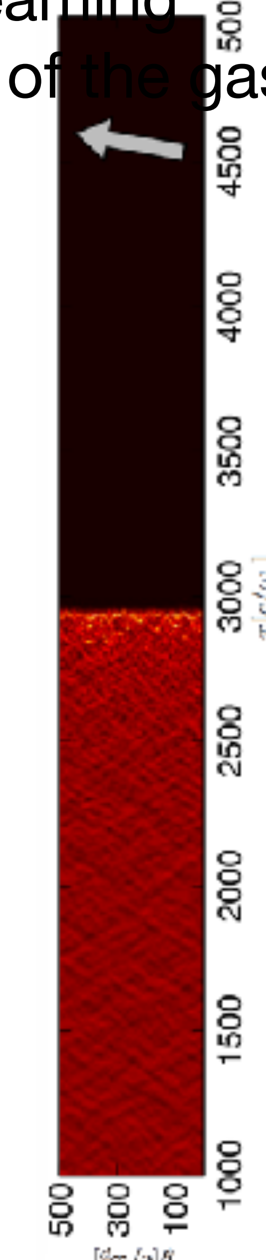






Caprioli & Spitkovsky 14

Shock finder

Candidate shock cells must verify:

$$\nabla.\vec{u} < 0$$

$$\nabla.\vec{u} \text{ is a local minimum along } \nabla T$$

$$\nabla T.\nabla S > 0$$

$$\mathcal{M} > \mathcal{M}_{\rm min} \simeq 1.5$$

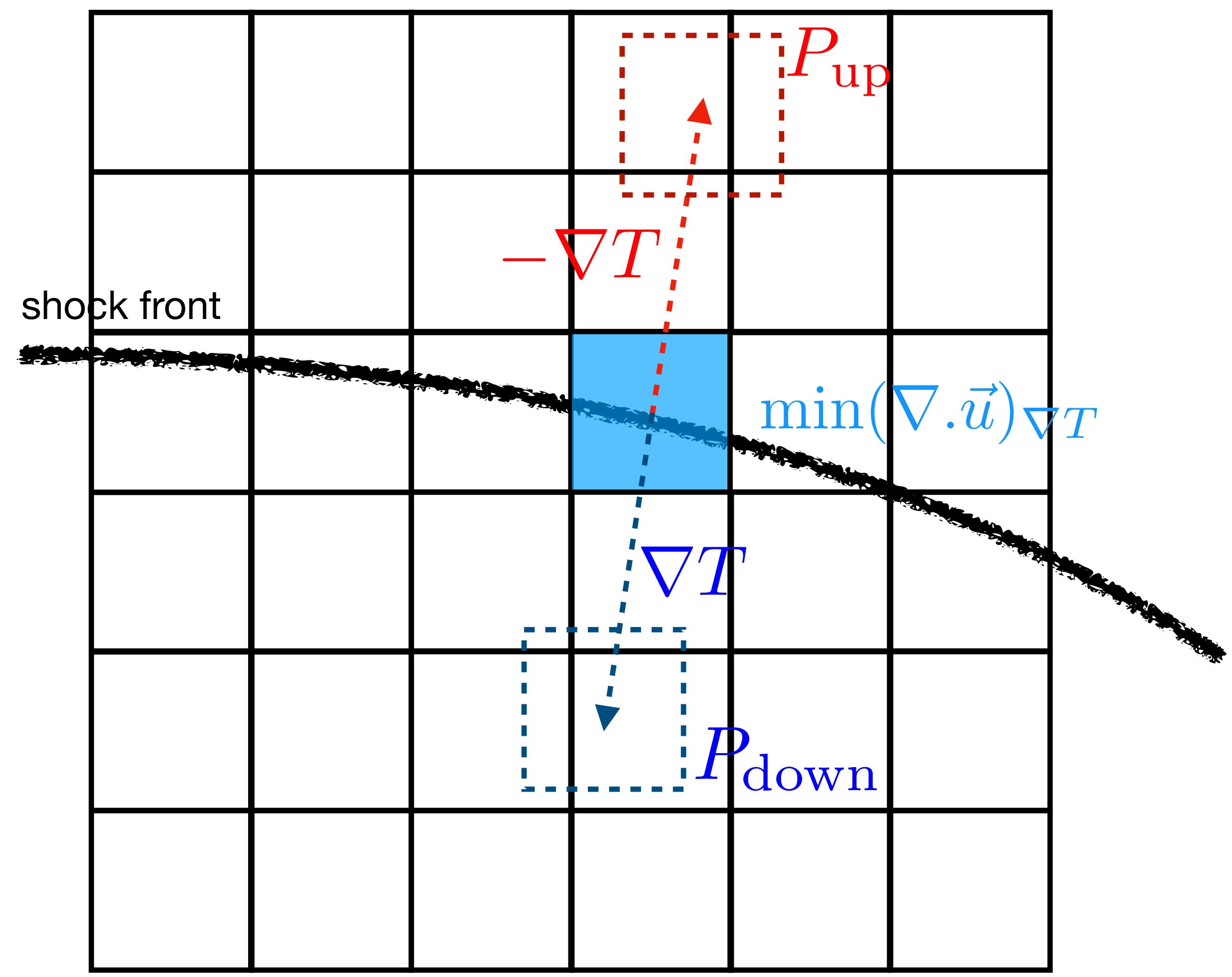
$$\mathcal{M} = \left\{ \frac{\gamma_{\rm e}+1}{2\gamma_{\rm e}} \left(\frac{\gamma_{\rm e}-1}{\gamma_{\rm e}+1} + \frac{P_{\rm down}}{P_{\rm up}} \right) \right\}^{1/2}$$

$$\gamma_{\rm e} = \frac{\gamma P_{\rm th} + \gamma_{\rm CR} P_{\rm CR}}{P}$$

Fully exploit the fact that a cell always have its neighboring octs in its own CPU domain (fast and easy access)

=> can search up to two cells distance in a symmetric way (upstream and downstream)

Do cloud-in-cell interpolation of upstream and downstream quantities along the normal to the shock $\pm\nabla T$



Shock finder

CR injection flux:

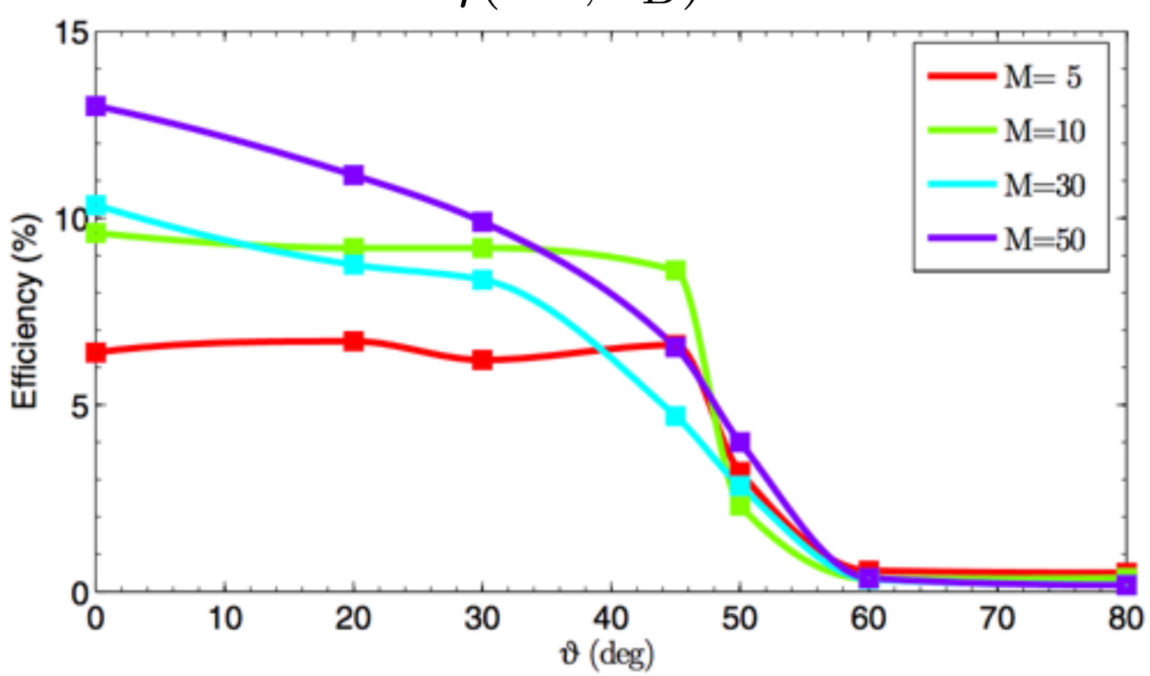
$$\phi_{\rm CR} = \eta(\mathcal{M}, \theta_{\rm B}) e_{\rm diss} \mathcal{M} c_{\rm s,up} / R_{\rm comp}$$

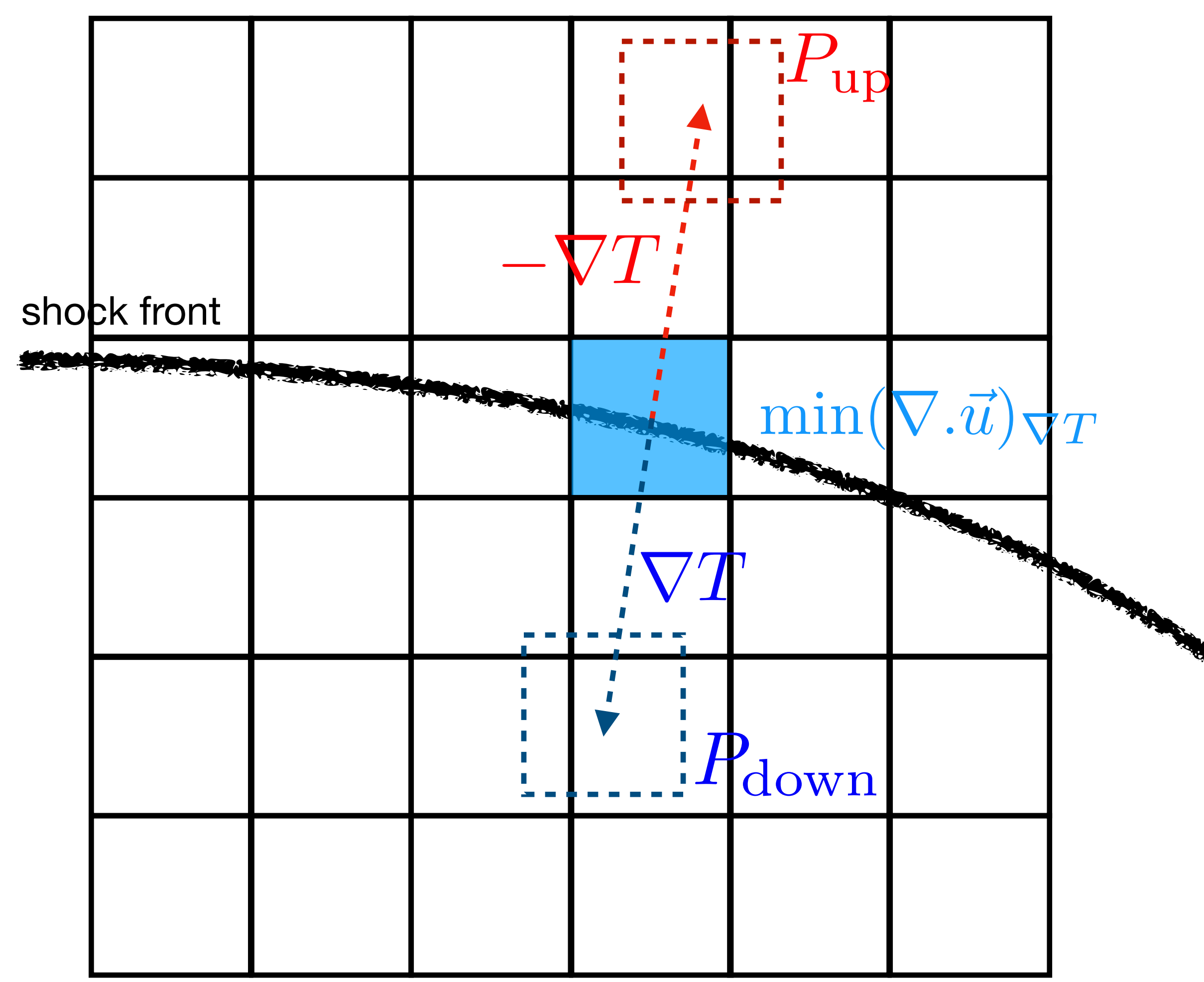
$$\Delta e_{\rm CR} = \phi_{\rm CR} \Delta t / \Delta x$$

$$R_{\rm comp} = \frac{\rho_{\rm down}}{\rho_{\rm up}}$$

$$e_{\rm diss} = e_{\rm th,down} - e_{\rm th,up} R_{\rm comp}^{\gamma}$$

$\eta(\mathcal{M}, \theta_{\rm B})$

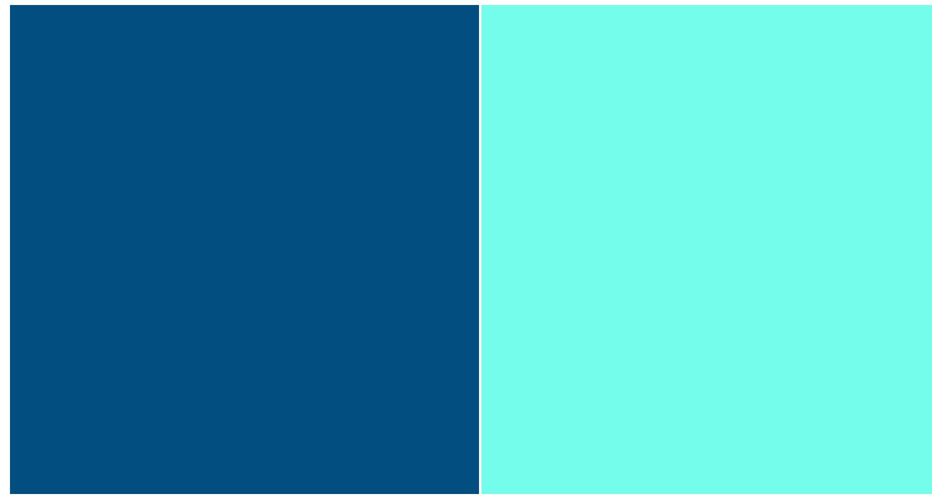




1D SOD shock tube test

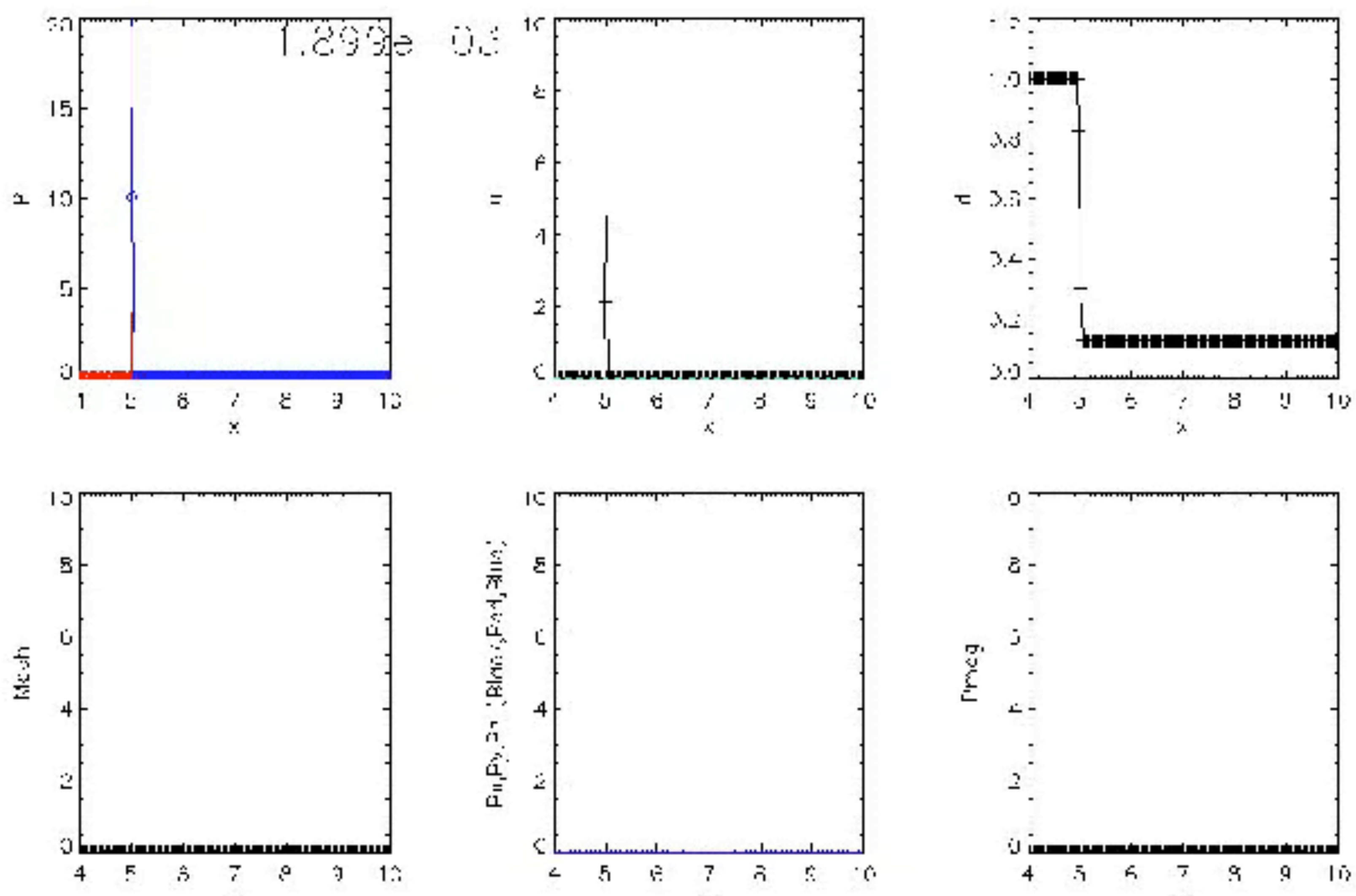
$$\gamma = 5/3 \quad \gamma_{\mathrm{CR}} = 4/3 \quad \mathcal{M} = 9.56 \quad \eta = 0.5$$

$$\rho_{\mathrm{L}} = 1$$
$$P_{\mathrm{th,L}} = 63.5$$
$$P_{\mathrm{CR,L}} = 0$$
$$u_{\mathrm{L}} = 0$$



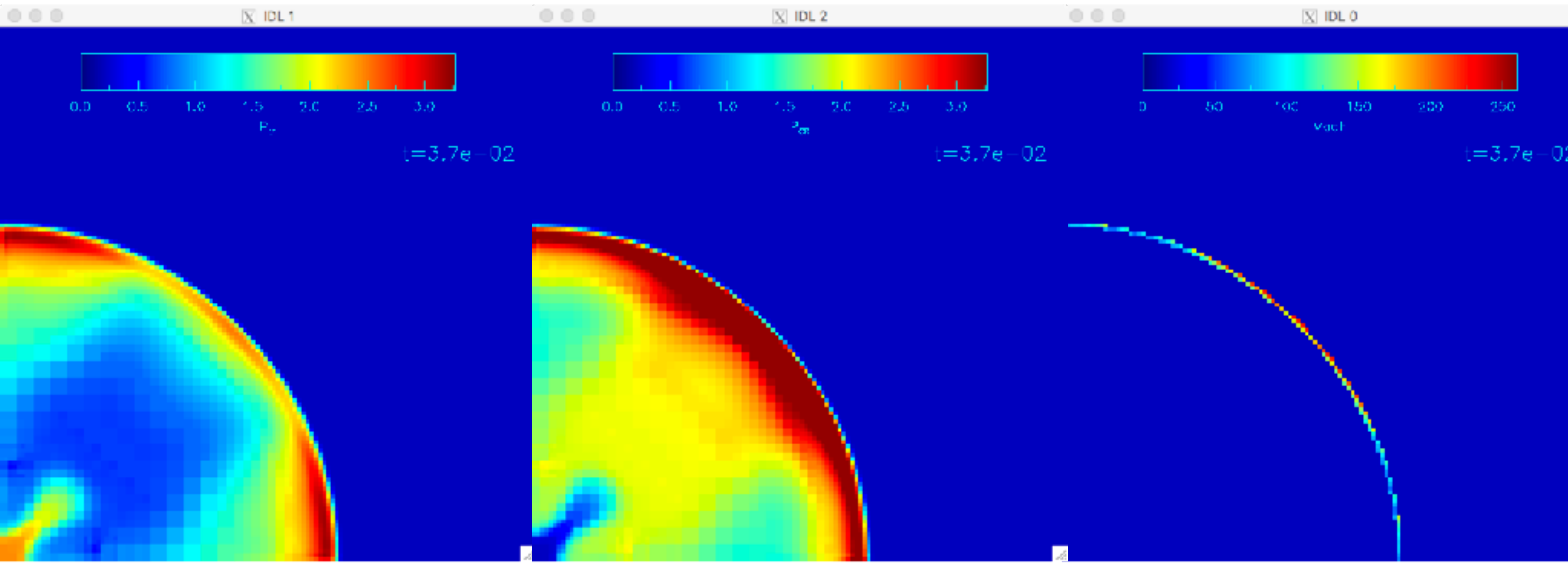
$$\rho_{\mathrm{R}} = 0.125$$
$$P_{\mathrm{th,R}} = 0.1$$
$$P_{\mathrm{CR,R}} = 0$$
$$u_{\mathrm{R}} = 0$$

Analytical solution of Pfrommer+17 for a composite thermal+CR gas



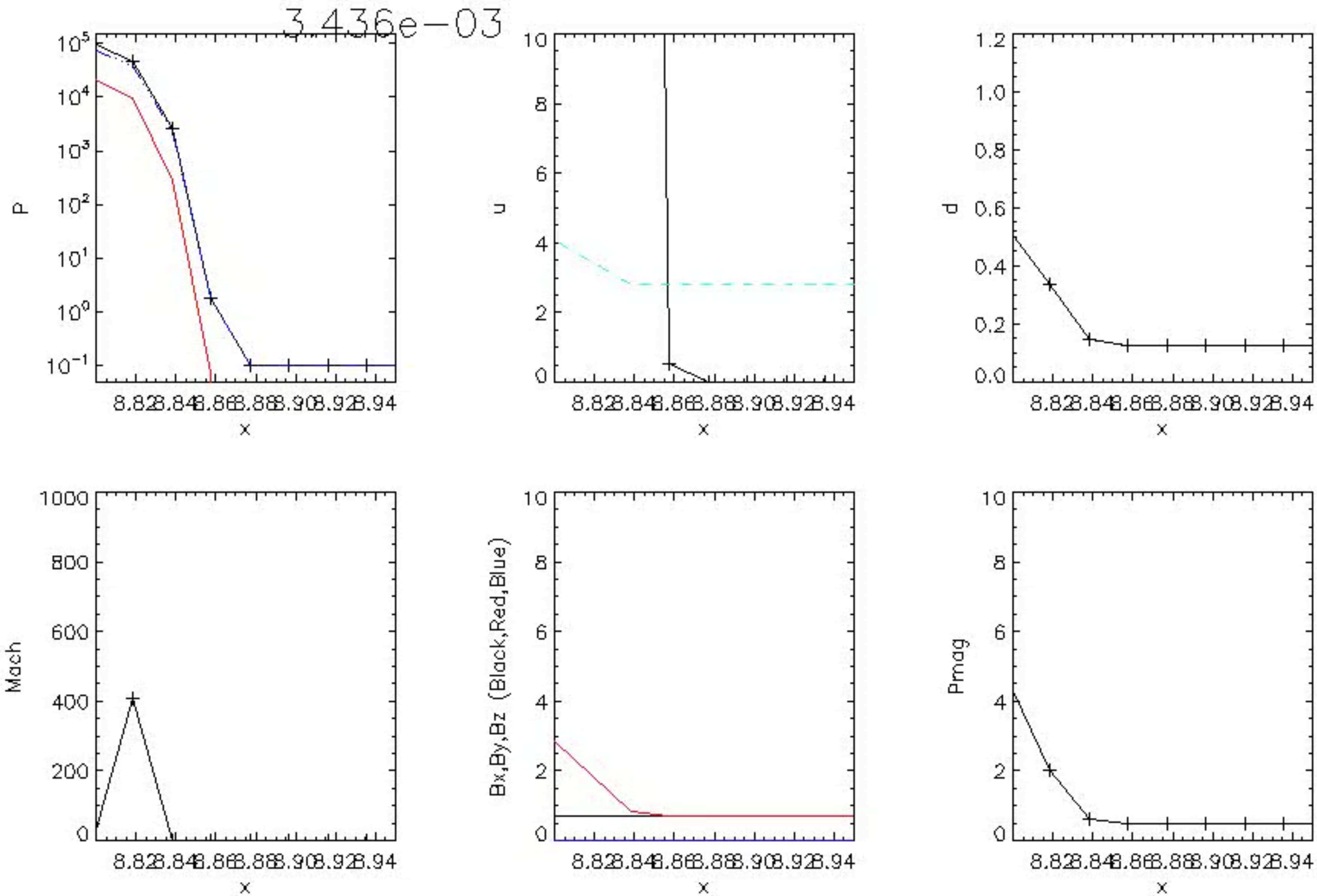
3D Sedov-Taylor explosion

The result should be spherically symmetric and it is not!



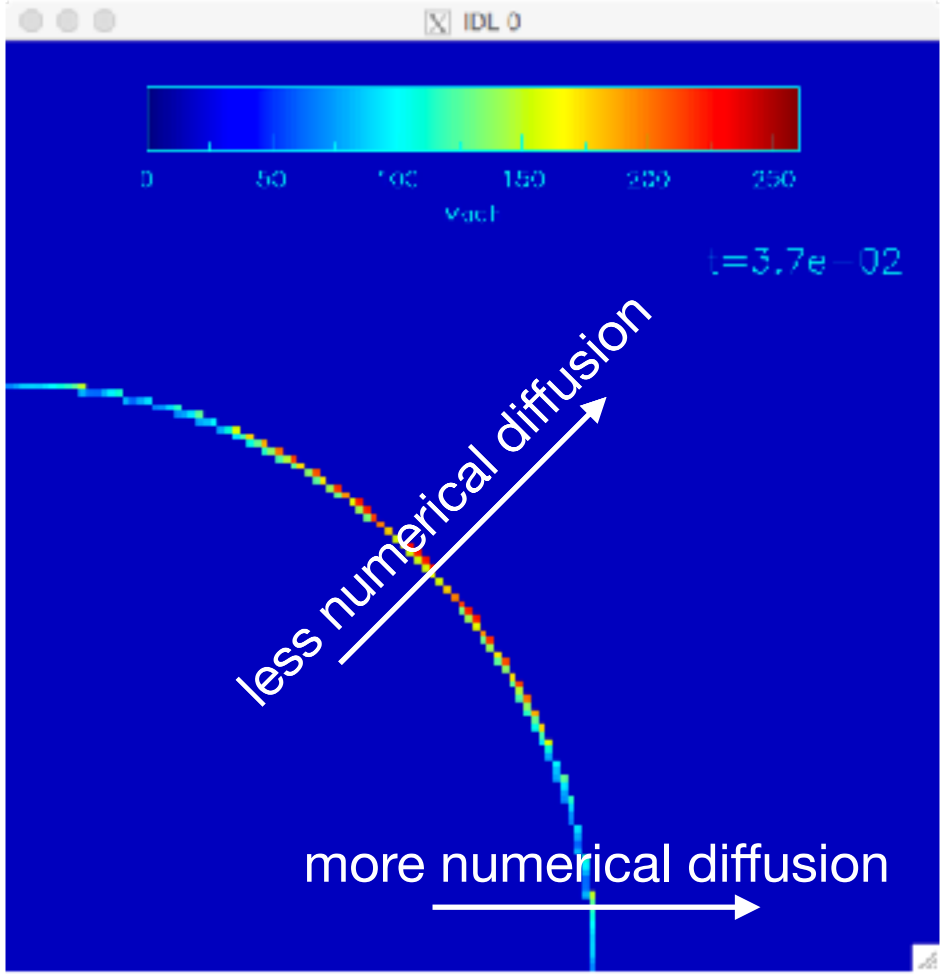
1D SOD shock tube test

$\mathcal{M} = 1000$



3D Sedov-Taylor explosion

The result should be spherically symmetric and it is not!



Solutions:

- Go further than 2 cells (1 oct) distance. Require MPI calls (inefficient?)
- Correct for underestimated high Mach numbers, as a function of shock direction w.r.t. the grid, on control tests (fishy?)